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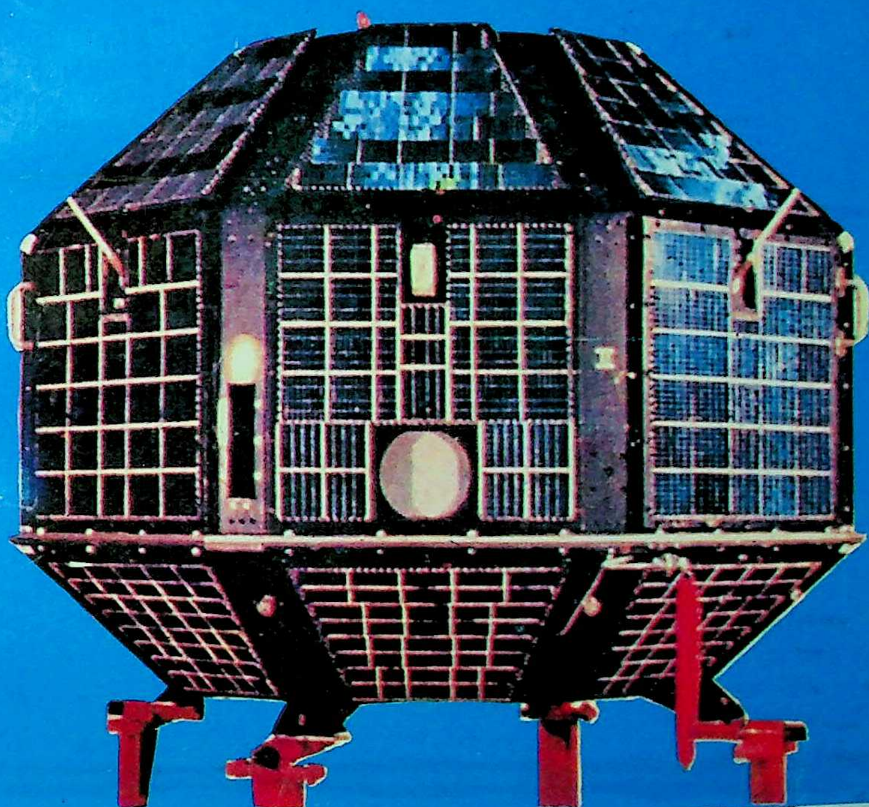

PRE-PROCEEDINGS
of the
International Conference
on
History of Mathematics

December 16-19, 1999

Department of Mathematics and Statistics

प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका
Journal of Natural & Physical Sciences

मुद्रक : किरण ऑफसैट प्रिंटिंग प्रेस, कनखल हरिद्वार। फोन: 415975

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No.F.2-M/99 1st December, 1999

Dear Dr. Dharm Pal,

The President of India, Shri K.R. Narayanan, is happy to know that the Gurukul Kangri Vishwavidyalaya, Hardwar is organising an International Conference to commemorate 1500 years of Aryabhatiyam of Aryabhata from 16th to 19th December, 1999.

The President extends his warm greetings and felicitations to the participants and the organisers and wishes the Conference all success.

With regards

Yours sincerely,

(S.N. Sahu)

Dr. Dharm Pal,
Vice Chancellor,
Gurukula Kangri Vishwavidyalaya
Hardwar 249 404,
Uttar Pradesh.

OM

Gurkula Kangri Vishwavidyalaya, Hardwar

Prof. Sher Singh
Visitor

☎ 011 - 6851718
011 - 6859234
011 - 6857711
Fax: 011 - 6522522

M-14, Saket
New Delhi-110 017

Date : 01-12-99

Dear Dr. Dharm Pal,

I am extremely happy to know that the Vishwavidyalaya is organizing an International Conference on History of Mathematics to commemorate 1500 years of Āryabhaṭiyam from 16th to 19th Dec. 99. This is the second conference, the Vishwavidyalaya is organizing within a year for highlighting the contribution of India in the development of the Science of Mathematics since antiquity. Mathematics undoubtedly enjoys a distinctive place in the development of Science, Technology and many other areas of human endeavour.

I pray for fruitful deliberations of the conference and for its success.

With warm greetings to the participants and organizers.

Yours sincerely,

Shersingh

(Sher Singh)

Dr. Dharm Pal
Patron, Organizing Committee and Vice-Chancellor
Gurukula Kangri Vishwavidyalaya
Hardwar

OM

Gurkula Kangri Vishwavidyalaya, Hardwar

Surya Dev
Chancellor

Phone: 3264129
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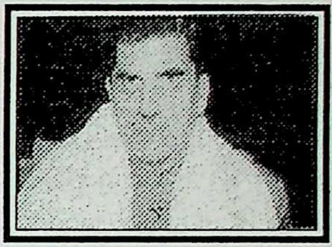
Maliwara, Delhi-110 006

Date : 02-12-1999



Dear Dr. Dharm Pal,

I am pleased to know that the Mathematics and Statistics Department of Gurukula Kangri Vishwavidyalaya is organizing an Internationl Conference on History of Mathematics. It is very much in tune to the Gurukuliya tradition that the conference is commemorating 1500 years of Āryabhaṭīyam. I extend a warm welcome to the delegates from India and abroad and wish a very successful conference.

Sincerely,

सूर्यदेव

Surya Dev
Chancellor, Gurukula Kangri Vishwavidyalaya

Dr. Dharm Pal
Vice-Chancellor
Gurukula Kangri Vishwavidyalaya
Hardwar

अरूण जेटली
ARUN JAITELY

सूचना और प्रसारण राज्य मंत्री
भारत सरकार
नई दिल्ली - 110 001

MINISTER OF STATE
INFORMATION & BROADCASTING
GOVERNMENT OF INDIA
NEW DELHI-110 001

03 DEC 1999

Message

I am glad to know that Gurukul Kangri Vishwavidyalaya is organising the International Conference on history of Mathematics to commemorate 1500 years of the composition of the Aryabhattiyam by the Mathematician Aryabhatta.

I extend my good wishes to the organisers and the participants on this occasion.

[ARUN JAITLEY]

Journal of Natural & Physical Sciences **Vol. 13 (1999), No. 1, 1-2**

स्वागत – भाषणम्

डॉ0 धर्मपाल:
संरक्षक: कुलपतिश्च
गुरुकुल कांगड़ी विश्वविद्यालयस्य

अमरहुतात्मना स्वामिश्रद्धानन्देन संस्थापिते गुरुकुल कांगड़ी विश्वविद्यालयस्य गणित एवं सांख्यिकी विभागे समायोज्यमाने गणित विद्वदन्ताराष्ट्रिय सम्मेलने गुरुकुलभुवमागता मान्या: कुलाधिपतय: श्रीमन्त: सूर्यदेवमहोदया:, मुख्यातिथय:, आयोजनसमितेरध्यक्षा: प्रो0 बी0एस0यादव महोदया:, आचार्य एवं उपकुलपतिपदे शोभमाना: प्रो0 वेदप्रकाशशास्त्रिण:, सर्वे संकायाध्यक्षा:, विभागाध्यक्षा:, प्राध्यापका:, कुलसचिवा: प्रो0 एस0एन0 सिंह महोदया:, भारतीय एवं विदेशीय विश्वविद्यालयेभ्य: शोधसंस्थानेभ्य: समागता: गणित विद्यापारङ्गता: विद्वांस:, गणित एवं सांख्यिकी विभागाध्यक्षा: डॉ0 वीरेन्द्र अरोड़ा महाभागा:, सम्मेलनस्य निदेशका विज्ञान महाविद्यालयस्य प्राचार्या: प्रो0 एस0एल0 सिंह महोदया:, सर्वे विद्वांसश्च।

अद्य युगपदेव समागतान् भवतो विलोक्य सर्वेषां गुरुकुलवासिनां मनांसि सानन्दानि सन्ति। विद्वांस: !

आर्यावर्तस्य विद्वद्भिर्वेदानधीत्य तदङ्गानि चाधीत्य महज्ज्ञानमर्जितम्। भारतवासिभिर्विद्यानां संख्यान विषये चतुर्दशत्वं स्वीकृतम्। चतुर्दशविद्यानिष्णाता मनस्विनो भवन्ति महान्त:। विद्यानां चतुर्दशत्वमित्थं समुदीरितम् –

अङ्गानि वेदाश्चत्वारो मीमांसा न्यायविस्तर:।
धर्मशास्त्रं पुराणञ्च विद्या ह्येताश्चतुर्दश।।

वेदानां परिपूर्णं ज्ञानं वेदाङ्गानामध्ययनेन भवति। तानि च वेदाङ्गानि षड् भवन्ति/यथा –

छन्द: पादौ तु वेदस्य हस्तौ कल्पोऽथ पठ्यते
ज्योतिषामयनं चक्षुर्निरुक्तं श्रोत्रमुच्यते।
शिक्षाघ्राणं तु वेदस्य मुखं व्याकरणं स्मृतम्
तस्मात्साङ्गान्यधीत्यैव ब्रह्मलोके महीयते।।

भारतवर्षस्य महर्षिभिर्ब्रह्मलोककामनया अङ्गानामध्ययनं तथाध्यापनं कृत्वा महद्यशः समर्जितम्। समये समये मनस्विभिरेकमेकं वेदाङ्गमादाय महता परिश्रमेण ग्रन्था निर्मिताः। तस्मिन्नेव क्रमे आर्यभटाचार्येण आर्यभटीयं ग्रन्थं निर्माय ज्योतिषशास्त्रस्य विस्तारः कृतः। ज्योतिषशास्त्रे आर्यभटीयस्य तदेव स्थानं विद्यते यत् स्थानं समुद्रे रत्नस्य। आर्यभटीये सर्वं ज्योतिर्विज्ञानं कारिका माध्यमेन लेखकेन समुन्मेषितम्। **''अर्थप्रदर्शन कारिका कारिका''** इति विद्यते कारिका लक्षणम् तथापि सामान्य जनानां कृते कारिकाणां तत्त्वावबोधाय भाष्यं व्याख्यानं वा समपेक्ष्यते। आर्यभटीयस्य कारिकाणां भाष्यं गार्ग्यकेरल नीलकण्ठ सोमसुत्व नाम धारिणा ज्योतिषशास्त्र पारङ्गतेन विदुषा विहितम्। आर्यभटाचार्येण आर्यभटीये स्वकीये ग्रन्थे समग्रस्य ज्योतिषशास्त्रस्य तत्त्वजातं संक्षिप्त रूपेण प्रतिपादितम्। सम्पूर्णोऽयं ग्रन्थः पादचतुष्टयी परिमितो विद्यते। अस्मिन् ग्रन्थे ग्रन्थ कर्त्रा सर्वत्र चार्या छन्दसः प्रयोगः कृतः। विदुषां प्रमोदकरमिदं वृत्तं यदार्यभटाचार्येण आर्यभटीये सर्वत्र आर्या प्रोद्गीता। पादचतुष्टयी परिमितेऽस्मिन् ग्रन्थे प्रथमे गीतिका पादे त्रयोदश आर्याः सन्ति गणित पादे त्रयस्त्रिंशत् आर्याःसन्ति कालक्रिया पादे पञ्चविंशति आर्याः सन्ति गोलपादे च पञ्चाशत् आर्याः सन्ति एवं सर्वा मिलित्वा सम्पूर्णे ग्रन्थे एकविंशत्युत्तरशत संख्यका आर्याः सन्ति। एतास्वेवानधिकसंख्यासु निगदितास्वार्यासु सकलं ज्योतिःशास्त्रं प्रभासते। आर्यभटीयस्य भाष्यं भाष्यकर्त्रा त्रिषु सम्पुटेषु कृतम्। एतेन भाष्येण ग्रन्थस्यास्य महती कीर्तिः प्रसृता। प्रस्तुतभाष्यात् प्राक् ग्रन्थोऽयं पाठकानां नासीत् सुखबोध्यः। परं सम्प्रति सभाष्यं ग्रन्थमधीत्य तद्गतं तत्त्वजातं निर्विलम्बतया अनायासेनैवावगच्छन्ति गणितविद्यानुरागिणः। ग्रन्थकर्त्रा ग्रन्थोऽयं वैदिक परम्परा विवृध्द्यर्थमेव निर्मितः। अमुं ग्रन्थमधीत्य सहजतयैव ज्ञायते यत् प्राक् काले गणितविद्या भारते व्योम्नि भानुरिव भासते स्म।

सम्प्रति गुरुकुल कांगड़ी विश्वविद्यालये गणितविभागे प्रायशः सम्मेलनानि क्रियन्ते येषु सम्मेलनेषु गणितविद्याप्रसङ्गः परिवर्धते। एवंविधेषु सम्मेलनेषु वैदिक दृष्ट्या शोधपत्राणि पठ्यन्ते। सम्प्रति वर्तमानेऽपि सम्मेलने गणित विद्याविशारदा वैदिक गणितविज्ञानमधिकृत्य शोध-पत्राणि पठिष्यन्ति। एतदर्थमहं विश्वविद्यालयपक्षतः सर्वेषामायोजकानां समागतविदुषां च हृदयेनाभ्युदयं कामये। अत्रागतानां समेषां सारस्वत पुत्राणां हार्दिकमभिनन्दनं करोमि।

Journal of Natural & Physical Sciences Vol. 13 (1999), No. 1, 3-4

वैदिक साहित्ये कालगणना

वेदप्रकाशः शास्त्री
आचार्य उपकुलपतिश्च
गुरुकुल कांगड़ी विश्वविद्यालयस्य

सकलमपि जगत् कालाश्रितं विद्यते, कालं विना किमपि जागतिकं कर्म न विद्यते। काल गणना सूर्यमाधारीकृत्य क्रियते। कालवादकः कालनियामको वा सूर्य एवास्ति नास्त्यत्र संशयः। वैदिक साहित्ये निमेषादारभ्य युगपर्यन्तं यावत् कालपरिगणना कृता युगाच्चारभ्य सृष्ट्यन्तं प्रलयं यावत् कालस्य सूक्ष्मातिसूक्ष्मो भागो भवति निमेषः। कालविद्भिःकालगणना सम्प्रति प्रथमतः सेकिण्ड नाम्ना क्रियते परं प्राक्तन ऋषिभि-र्निमेषादारम्य कालगणना कृता। निमेषस्तु सेकिण्डपदवाच्यात् कालादतिसूक्ष्मः। मनुना मनुस्मृतौ यथा कालगणना संदिष्टा साऽऽतीव प्राचीना सूक्ष्मान्वेषणयोग्या च। यथा-

निमेषा दश चाष्टौ च काष्ठा त्रिंशत्तु ताः कला
त्रिंशत्कला मुहूर्तः स्यादहोरात्रं तु तावतः।।

कोऽयं निमेष इत्युद्गतायां जिज्ञासायाम् अन्वर्थकमुक्तावली व्याख्याकारः कथयति-अक्षिपक्ष्मणोः स्वाभाविकस्य उन्मेषस्य सहकारी निमेषः। तेषामष्टादशनिमेषाणां काष्ठा नामकः कालो भवति। त्रिंशच्च काष्ठाः कलासंज्ञकः त्रिंशत्कलाः मुहूर्ताख्यः कालः। तावत् त्रिशंन् मुहूर्तान् अहोरात्रं कालं विद्यादिति। एकाकी चरन् सूर्य एव अहोरात्रजनयिता अस्ति। यथा चोक्तम्-

अहोरात्रे विभजते सूर्यो मानुषदैविके।
रात्रिः स्वप्नाय भूतानां चेष्टायै कर्मणामहः।।

मानुषदैवसम्बन्धिनौ दिनरात्रिकालावादित्यः पृथक् करोति। तयोर्मध्ये भूतानां स्वप्नार्थं रात्रिर्भवति कर्मानुष्ठानार्थं च दिनमिति। एवं मानुषाणामहोरात्रकालगणनां निगद्य पितृणामहोरात्रगणनायां कथयति यत् मानुषाणां मासपरिमितः कालः पितृणां त्वहोरात्रे भवतः। तत्र पक्ष द्वयेन विभागः। कर्मानुष्ठानाय पूर्वपक्षोऽहः। स्वापार्थं शुक्लपक्षो रात्रिः। यथा-

पित्र्ये रात्र्यहनी मासः प्रविभागस्तु पक्षयोः।
कर्मचेष्टास्वहः कृष्णः शुक्लः स्वप्नाय शर्वरी।।

मानुषाणां पितृणाञ्चाहोरात्रकालं कथयित्वा देवानामहोरात्रकालविषये निगदति यत् मानुषाणां यदेकं वर्षं भवति तत्तु देवानाम् अहोरात्रमितः कालो भवति तत्राप्ययं विभागः। नराणामुद्गयनं देवानामहः। तत्र प्रायेण दैवकर्मणामनुष्ठानं, दक्षिणायनं तु रात्रिः।

दैवे रात्र्यहनी वर्षं प्रविभागस्तयोः पुनः।
अहस्तत्रोद्गयनं रात्रिः स्याद्दक्षिणायनम्।।

यं सूर्यमाधारी कृत्य मानुषाणां पितॄणां दैवतानां कालः परिगणितः तत्सूर्यपरिज्ञानेनैव ब्रह्मणो दिनस्य निशायाश्च कालविषये विचार प्रकटितः।

महर्षिदयानन्दैः सृष्टिक्रमे कालगणनासन्दर्भे यदुदीरितं तदैव विलिख्यते–

''अस्यां वर्तमानायां सृष्टौ वैवस्वतस्य सप्तमस्यास्य मन्वन्तरस्येदानीं वर्त्तमानत्वादस्मात्पूर्वं षण्णां मन्वन्तराणां व्यतीतत्वाच्चेति। तद्यथा–स्वायम्भुवः, स्वारोचिष, औत्तमिस्तामसो, रैवतश्चाक्षुषो: वैवस्वतश्चेति सप्तैते मनवस्तथा सावर्ण्याद्या आगामिनः सप्त चैते मिलित्वा चतुर्दशैव भवन्ति। तत्रैक सप्ततिश्चातुर्युगानि ह्येकैकस्य मनो परिमाणं भवति। ते चैकस्मिन् ब्राह्मदिने चतुर्दश–भुक्त भोगाः भवन्ति। एकसहस्त्रं 1000 चातुर्युगानि ब्राह्मदिनस्य परिमाणं भवति। ब्राह्म्या रात्रेरपि तावदेव परिमाणं विज्ञेयम्। सृष्टेर्वर्तमानस्य दिन संज्ञास्ति प्रलयस्य च रात्रिसंज्ञेति। अस्मिन् ब्राह्मदिने षट् मनवस्तु व्यतीताः, सप्तमस्य वर्तमानस्य मनोरष्टाविंशतितमोऽयं कलिर्वर्तते।''

एवं प्रकारेण महर्षिणा समग्रस्य सृष्टिक्रमस्य कालः परिगणितः। अस्यां कालगणनायां स्वयं मनुना यल्लिखितं तत्सर्वं वेदमाश्रित्यैव लिखितम्–यथा

ब्राह्मस्य तु क्षपाहस्य यत्प्रमाणं समासतः।
एकैकशो युगानां तु क्रमशस्तन्निबोधत।।
चत्वार्य्याहुः सहस्राणि वर्षाणां तु कृतं युगम्।
तस्य तावच्छती सन्ध्या सन्ध्यांशश्च तथाविधः।।
इतरेषु ससन्ध्येषु ससन्ध्यांशेषु च त्रिषु।
एकापायेन वर्तन्ते सहस्राणि शतानि च।।
यदेतत् परिसंख्यातमादावेव चतुर्युगम्।
एतद् द्वादशसाहस्रं देवानां युगमुच्यते।।
दैविकानां युगानां तु सहस्त्रं परिसंख्यम।
ब्राह्ममेकमहर्ज्ञेयं तावती रात्रिरेव वा।।
तद् वै युगसहस्रान्तं ब्राह्मं पुण्यमहर्विदुः
रात्रीं तावतीमेव तेऽहोरात्रविदो जनाः।।
यत्प्राग् द्वादशसाहस्रमुदितं दैविकं युगम्।
तदेकसप्ततिगुणं मन्वन्तरमिहोच्यते।।
मन्वन्तराण्यसंख्यानि सृष्टिः संहार एव च।
क्रीडन्निवैतत्कुरुते परमेष्ठी पुन पुनः।।

एवं प्रकारेण वैदिक साहित्ये सकला काल परिगणना कृता। भारतीयानां कालगणनाक्रमो नूनमेवादिमः। सूर्यसिद्धान्तमाश्रित्य भारतस्य मनीषिभिरेक एव नापितु बहुशो ग्रन्था निर्मिताः।

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FROM THE ORGANIZING SECRETARY - S. L Singh

Āryabhaṭa (b.476 AD), generally known as the father of ancient Indian mathematics, composed his Āryabhaṭīyam at the age of 23 years. This legendary composition was designed to cater the need of a text book of astronomy and mathematics. According to Āryabhata, the Āryabhaṭīyam is essentially based on Vedas.

This is an overwhelming occasion that we are commemorating 1500 years of Āryabhaṭīyam by organizing an International Conference on History of Mathematics. The idea of the conference was essentially conceived by Professor B. S. Yadav and Professor Aruna Kapur during the last December when they were attending the 64th Indian Mathematical Society Conference organized by this Vishwavidyalaya at Hardwar. It was immediately supported by Mr. N. D. Malhotra and Dr. Virendra Arora. In an independent communication, Mr. Sailesh Dasgupta suggested me to commemorate 1500 years of Āryabhaṭīyam. My health and future plans were not permitting me to say "yes", and my respect to Professor Yadav & other friends and of course my love for Āryabhaṭīyam & Vedic knowledge did not permit me to say "no". So, with an indecision, I approached our Vice Chancellor Dr. Dharm Pal Ji with a mild hope in the heart of heart that he would say "no". But being a better lover of Vedic Sciences and traditions, he listened carefully the whole plan of organization and was quick to give his consent and to assure Vishwavidyalaya's support. I had no option but to go ahead. Indeed, we are able to organize this conference because of his dynamic leadership and valuable cooperation from the administration, viz., Registrar Professor S. N. Singh and Finance Officer Mr. J. S. Gupta.

On behalf of the organizing committee and my own behalf, I extend a very warm welcome to the delegates from India & abroad, our distinguished guests, friends, colleagues, ladies and gentlemen. It is a matter of great pleasure and honour that this historic occasion is being graced by hon'ble Visitor Professor Sher Singh, hon'ble Chancellor Shri Surya Dev Ji and respected Chief Guest. I extend a very warm welcome to them.

I am very happy to express my heart felt indebtedness to the organizing committee patron Dr. Dharm Pal Ji, Chairman Professor B. S. Yadav, Acharya Ved Prakash, Dean Vinod Kumar, Dr. Virendra Arora, Dean S. C. Dhamija, Dr. Prabhakar Pradhan, other members of the organizing committee, my colleagues, friends, saints and saintly persons from inside & outside Vishwavidyalya, especially from Hardwar, Rishikesh and New Tehri. With the risk of missing many names, I respectfully acknowledge assistance, cooperation and blessings from (off-campus, in chronological order of contacting them): Dr. Raj Kumar Singh Rawat, General Manager, Gurukula Pharmacy; Professor B. S. Yadav, Delhi, Professor Aruna Kapur, Jamia, Delhi; Mr. N. D. Malhotra, Hardwar; Professor P. K. Jain, Delhi, Dr. Man Mohan, Delhi; Mr. S. C. Sharma, General Manager, THDC, New Tehri; Mr. K. L. Kashyap & Mr. Gajanand Goyal of Mandi Govind Garh Charitable Trust Dharmashala, Hardwar; Mr. I. D. Joshi & Mr. D. K. Varshaney of Rishikesh; Millennium Technologoies Ltd., New Delhi; Svami Chidanand Muni Ji, President, Parmarth Niketan, Rishikesh; Shri Bhaktikyoga Svami Ji of New Madhuban Ashram, Rishikesh; Shri Sudhir Gupta and Messers Kiran Printers of Kankhal-Hardwar.

The organizing committee thanks the CSIR, New Delhi and the Indian National Science Academy, New Delhi for their financial assistance. I shall be failing in my duty if do not thank the National Board of Higher Mathematics, Mumbai for communicating its decision in time to give "zero finanical assistance". It is worth mentioning and, indeed, it shows the suitability of our programme and high spirits of a matured mathematician of international repute that Dr. Marry B. Coonce, Professor Emeritus of USA has donated US$ 200.00 for our conference. We thankfully acknowledge his assistance which has come to us without any appeal for support.

Members of the organizing committee have played their roles with dedication and best of their capability. I personally owe to them. Mr. Lakshami P. Purohit, JRF (CSIR, Physics) deserves high appreciation for his valuable cooperation.

Teachers and non-teaching staff, whom we approached, extended their full cooperation and help in this penrose venture. The registery and finance department of our Vishwavidyalaya deserve appreciation for not applying much brakes.

Once again, I extend warm greetings to distinguished guests, colleagues, friends, students and others associated with this conference. ***Jai Ganita.***

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A BIRD VIEW OF VEDIC MATHEMATICS IN GKV

Virendra Arora

Head, Department of Mathematics and Statistics, GKV, Haridwar

The Gurukula Kangri Vishvavidyalya was founded in 1902 by Swami Shradhanand Ji Maharaj (1856-1926) to have an alternate system of education over that of British pattern English medium education. The emphasis was on "Guru-Shishya" relationship which helps more in character building of students and over all development.

Swami Ji has a vision of having intraction between Vedic and Modern Sciences which is a vital component of new education system.

The education of Science Subjects started in GKV in 1910. Mathematics, Chemistry, Physics, Biology, were a few Subjects taught in Hindi medium.

Separate Science College was founded in 1921 and its building was inaugurated by the (then) Prime Minister Pandit Jawaherlal Nehru on 1.8.1958. Post graduate teaching of mathematics could start in 1964. At this time Gurukula Kangri was also having under graduate department of mathematics in the College of Science.

A post of professor of mathematics was created by the U.G.C. under development plan in 1984 and Prof. S.L. Singh was appointed professor and joined the dept. in 1985. Subsequently the two department were merged in July 1985. Doctoral level research activities in history of mathematics were initiated in the department by Prof. S. L. Singh. The first Ph.D enrollment in history of mathematics was done under his supervision at the close of 80's.

Indeed this was a good beginning of research activities in GKV. Now almost every department of Science College is having a few master courses or doctoral level research work in Vedic Sciences. Department of mathematics has already organized three national level symposium on mathematics and Vedic

mathematics in 1989, 1995 and 1998 and The 64th Indian Mathematical Society Conference (1998) under the leadership of Prof. S.L. Singh. Now the department of mathematics is honoured to organize this commemorative International Conference.

The following table gives a glimpse of research level activities in Vedic Mathematics.

	Name	Title & Year of Ph.D Thesis	Supervisor
1.	Ramesh Chand	A Study of Sidhanta Siromoni (1990)	S. L. Singh
2.	Vinod Mishra	Vedic geometry & its applications (1997)	S. L. Singh
3.	N.B. Singh	Indeterminate analysis & its applications (1997)	S. L. Singh
4.	Vivek Goel	A Study of Ganitatilaka (1997)	Virendra Arora
5.	Nidhi Handa	Katyayana Sulba Sutra & modern mathematical Interpreition of its sutras (Enrolled for Ph.D)	Virendra Arora
6.	Ghan Shyam Singh	Ghanirveda & mathematics of vedic Traditions (Enrolled for Ph.D)	Virendra Arora
7.	Dhirendra Joshi	Vedic Mathematics-formulae & mathematical incidences in ancient Indian epics (Applied for Enrollment for Ph.D)	S. L. Singh
8.	Mohd. Mustafizur Rehaman	A Development of Astronomy in India (Enrolled for Ph.D)	V. K. Sharma

Although Professor S.L. Singh's primary interest is Nonlinear Analysis but his first love is "Līlāvatī", and therefore we are happy that his "first love" has given an impetus for research level activities in Vedic Mathematics. He will be remembered for his contribution to create an excellent environment for the study of Vedic Mathematics in GKV. The department of Mathematcs and statistics is undoubtedly indebted for his multidimensional contribution compatible to the Gurukuliya Traditions.

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वैदिक वाङ्मय में गणितशास्त्र के मूलतत्त्व

पद्मश्री डॉ0 कपिलदेव द्विवेदी

निदेशक, विश्वभारती अनुसंधान परिषद, ज्ञानपुर (भदोही)

गणित का महत्त्व

गणित समस्त विज्ञान का मूल है। गणित ही सृष्टि - रचना के मूल में है। संसार की प्रत्येक वस्तु किसी नियम से बद्ध है, उसमें कोई क्रम है। उस नियम और क्रम का ज्ञान गणित का विषय है। सृष्टि की प्रत्येक वस्तु में गति है। गति का संबन्ध गणन से है। यह गणना गणित का विषय है। सूर्य, चन्द्र, नक्षत्र, ग्रह एवं पृथिवी की गति के ज्ञान से ही सूर्योदय, सूर्यास्त, सूर्य - ग्रहण, भू - परिक्रमा आदि का ज्ञान होता है। पूरा ज्योतिष - शास्त्र गणित पर निर्भर है। स्थान और समय का निर्धारण गणित के आधार पर ही होता है। गति का निरन्तरता का नाम समय (Time) है और गति का चतुर्दिक् प्रसार ही स्थान (Space) है। इनके ज्ञान के लिए गणित की आवश्यकता होती है। गणित के द्वारा गति का आकलन किया जाता है, अतः गणित विज्ञान की आधारशिला है।

वेदांग - ज्योतिष में गणितशास्त्र का महत्त्व बताते हुए कहा गया है कि जिस प्रकार मयूरों की शिखाएँ और नागों (सर्पों) की मणियाँ सर्वोच्च स्थान पर रहती हैं, उसी प्रकार सारे वेदांगों में गणित का स्थान सर्वोपरि है।

यथा शिखा मयूराणां नागानां मणयो यथा।

तद्वद् वेदांगशास्त्राणां गणितं मूर्धनि स्थितम्। वेदांगज्योतिष (याजुष) 4

प्रसिद्ध जैन गणितज्ञ महावीराचार्य (लगभग 850 ई0) ने अपने ग्रन्थ 'गणितसारसंग्रह' (अध्याय 1, श्लोक 9 - 19) में कहा है कि अधिक गुणगान से क्या लाभ। इस चराचन जगत् में ऐसी कोई वस्तु नहीं है, जिसके मूल में गणित न हो। गणित ज्ञान और विज्ञान की सभी शाखाओं का आधार है।

बहुभिर्विप्रलापैः किं त्रैलोक्ये सचराचरे।

यत् किंचिद् वस्तु तत् सर्वं गणितेन विना न हि।।

वेदों में छन्द - रचना के मूल में गणित है। इसी आधार पर गायत्री (8.8 । 8 = 24 वर्ण), अनुष्टुप् (8,8 । 8.8 = 32 वर्ण), त्रिष्टुप् (11,11 । 11,11 = 44 वर्ण), जगती (12,12 । 12,12 = 48 वर्ण) आदि विभिन्न छन्दों की विशेषताओं की सृष्टि हुई है।

गणितशास्त्र का उद्भव

वेदों के अध्ययन से ज्ञात होता है कि उनमें गणितशास्त्र से संबद्ध पर्याप्त सामग्री उपलब्ध है। उनमें एक संख्या से लेकर परार्ध संख्या तक का उल्लेख है। उनमें 10 संख्या का महत्त्व, उसके गुणन, स्थानमान तथा भाग आदि का वर्णन है। इनका वर्णन आगे किया जाएगा।

वेदों में गणित शब्द का उल्लेख नहीं है। कुछ अन्य शब्द मिलते हैं, जिनसे ज्ञात होता है कि गणना की विधि ज्ञात थी। यजुर्वेद (30.20) और तैत्तिरीय ब्राह्मण (3.4.15) में गणक (गणना करने वाला, ज्योतिषी) शब्द मिलता है। गणनासूचक गण, गणपति, गणश्रि, गणिन्, गण्य आदि शब्द ऋग्वेद और यजुर्वेद में अनेक मंत्रों में आये हैं।[1] ऋग्वेद में 'व्रातंव्रातम्, गणंगणम्' शब्द गणना के आधार पर किए गए समूहों या वर्गों के लिए हैं।[2] इसी प्रकार यजुर्वेद और अथर्ववेद में निधि, निधिपति, निधिपा शब्द कोष और कोषागार के अध्यक्ष के लिए हैं।[3] ये कोष की गणना करते थे। यजुर्वेद में वित्तध शब्द भी कोषागार के अध्यक्ष के लिए आया है।[4] यजुर्वेद में ज्योतिषी के लिए 'नक्षत्रदर्श' शब्द है ओर गणित-विद्या जानने के कारण उसके ज्ञान की प्रशंसा की गयी है।[5]

छान्दोग्य उपनिषद् में सर्वप्रथम गणितशास्त्र का 'राशिविद्या' और ज्योतिष का 'नक्षत्रविद्या' नाम से उल्लेख है।[6] सनत्कुमार के पूछने पर नारद ने बताया कि मैंने ये विद्याएँ पढ़ी हैं। उन विद्याओं में नारद ने चारों वेद, इतिहास, पुराण, ब्रह्मविद्या आदि के साथ राशिविद्या और नक्षत्रविद्या का भी उल्लेख किया है। राशिविद्या शब्द अंकगणित के लिए है और नक्षत्रविद्या ज्योतिष के लिए है। उक्त कथन से ज्ञात होता है कि अध्यात्म या पराविद्या के जिज्ञासु के लिए गणित ओर ज्योतिष का भी ज्ञान अपेक्षित है।

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1. गणानां त्वा गणपतिं हवामहे। ऋग्0 2.23.1, यजु0 23.19, तैत्ति0 सं0 2.3.14.3
   गण्या, ऋग्0 3.7.5 । गणश्रिभिः। ऋग्0 5.60.8
2. व्रातंव्रातं गणंगणम्। ऋग्0 3.26.6 । 5.53.11
3. निधीनां त्वा निधिपतिम्। यजु0 23.19 । अथर्व0 7.18.4 ।
   निधिपा, अथर्व0 12.3.34
4. वित्तधम्, यजु0 30.11
5. प्रज्ञानाय नक्षत्रदर्शम्। यजु0 30.10
6. स होवाच-ऋग्वेदं भगवाऽध्येमि ... राशिं दैवं निधिं ... नक्षत्रविद्याम् ... अध्येमि। छान्दोग्य उप0 7.1.2

अंकगणित के विषय : गणित के विभिन्न कार्यों के लिए सामान्य रूप से 'परिकर्म' (Fundamental operations) शब्द का प्रयोग होता था। ब्रह्मगुप्त ने 'ब्राह्मस्फुट-सिद्धान्त' में अंकगणित में 20 विषय और 8 व्यवहार सम्मिलित किए थे। 1. संकलित (जोड़) 2. व्यवकलित (घटाना), 3. गुणन (गुणा करना), 4. भागहार (भाग देना), 5. वर्ग (Square) 6. वर्गमूल (Squareroot) 7. घन (Cube) 8. घनमूल (Cuberoot), (9-13) पंचजाति (भिन्न के 5 प्रकार), 14. त्रैराशिक (Rule of three) आदि।

भास्कराचार्य ने लीलावती में अभिन्न-परिकर्माष्टक (पूर्ण संख्याओं के 8 परिकर्म) और भिन्नपरिकर्माष्टक (अपूर्ण संख्याओं के 8 परिकर्म) अंकगणित के विषय बताए हैं। दोनों में 8 परिकर्म ये हैं : 1. संकलन, संकलित (जोड़ना), 2. व्यवकलन, व्यवकलित (घटाना), 3. गुणन (गुणा करना), 4. भागहार (भाग देना), 5. वर्ग, 6. वर्गमूल, 7. घन, 8. घनमूल।

संख्यासूचक शब्द

यजुर्वेद, तैत्तिरीय संहिता, मैत्रायणी और काठक संहिताओं में 1 से परार्ध तक की संख्याओं के नाम मिलते हैं। इनमें से प्रत्येक अगली संख्या की 10 गुनी है। ये नाम हैं :

एक (1), दश (10), शत (100), सहस्र (1000), अयुत (10 हजार), नियुत (1 लाख), प्रयुत (10 लाख), अर्बुद (1 करोड़), न्यर्बुद (10 करोड़), समुद्र (1 अरब), मध्य (10 अरब), अन्त्य (1 खरब), परार्ध (10 खरब)। ऋग्वेद में एक, दश, शत (1.24.9) सहस्र (1.24.9) में शत, सहस्र, अयुत और न्यर्बुद तक संख्याएं दी हैं।

पंचविंश ब्राह्मण में न्यर्बुद तक तो यजुर्वेद वाली ही नामावली है। उसके बाद निखर्व, वाडव, अक्षिति आदि नाम हैं। शांखायन श्रौतसूत्र में न्यर्बुद के बाद निर्खव, समुद्र, सलिल, अन्त्य और अनन्त (10 Billions) आदि संख्याएँ दी गयी हैं।[2] इनमें से प्रत्येक अपने पूर्ववर्ती से 10 गुने हैं। अत: इन्हें 'दशगुणोत्तर संज्ञा' कहते हैं।[3]

संख्याओं का स्थानिक मान (Notational places) : क्रमश: स्थानमान की भावना का विकास हुआ। दशम-पद्धति पर संख्याओं का लिखना भारतवर्ष का विशेष आविष्कार है। आर्यभट प्रथम (सन् 499) ने आर्यभटीय (2.2) में लिखा है कि 'किसी लिखी हुई संख्या में एक-एक स्थान

1. एका च, दश च, शतं च, सहस्रं च, अयुतं च, नियुतं च, प्रयुतं च, अर्बुदं च, न्यर्बुदं च, समुद्रश्च, मध्यं च, अन्तश्च, परार्धश्च। यजु0 17.2। तैत्ति0 सं0 4.4.1 सं0 4.4.11; 7.2.20 । मैत्रा0 2.8.14 । काठक0 17.10.31
2. शांखा0 श्रौत0 15.11.4
3. दशगुणोत्तरं संज्ञा:। लीलावती 1.2

बाईं ओर हटने पर स्थानिक मान निम्नलिखित क्रम में 10 गुना बढ़ता जाता है। एक (इकाई), दश (दहाई), शत (सैकड़ा), सहस्र (हजार), अयुत (दस हजार), नियुत (लाख), प्रयुत (दस लाख), कोटि (करोड़), अर्बुद (10 करोड़) और वृन्द (अरब)।[4] इसमें अंक-संज्ञा के अर्थ में 'स्थान' शब्द का प्रयोग हुआ है। यह स्थानिक मान का सूचक है।

श्रीधर (750 ई0) ने अपने ग्रन्थ 'त्रिशतिका' में 18 स्थानों के नाम दिए हैं और इन्हें 'दशगुणाः संज्ञा' कहा है।[5] ये हैं एक, दश, शत, सहस्र, अयुत, नियुत, प्रयुत, कोटि, अर्बुद, अब्ज, खर्व, निखर्व, महासरोज, शंकु, सरितापति, अन्त्य, मध्य, परार्ध। भास्कर द्वितीय (1150 ई0) ने लीलावती ग्रन्थ में श्रीधर की ही नामावलि ली है।[6] केवल तीन स्थानों पर अन्तर किया है। दो स्थानों पर इनके पर्यायवाची शब्द दे दिए हैं। ये हैं महासरोज के स्थान पर महापद्म और सरितापति के स्थान पर जलधि शब्द तथा नियुत के स्थान पर लक्ष शब्द।

यजुर्वेद (17.2) के भाष्य में महीधर ने एक से परार्ध तक 18 संख्या-संज्ञाओं का उल्लेख किया है और कहा है कि न्यर्बुद के बाद खर्व, निखर्व, महापद्म और शंकु संज्ञाओं का भी परिगणन समझना चाहिए।[7] इस प्रकार 18 संख्या संज्ञाएँ ये हैं :

1. एक (1) $= 10^0$
2. दश (10) $= 10^1$
3. शत (100) $= 10^2$
4. सहस्र (1000) $= 10^3$
5. अयुत (10 हजार) $= 10^4$
6. नियुत (लक्ष) (1 लाख) $= 10^5$
7. प्रयुत (10 लाख) $= 10^6$
8. कोटि (1 करोड़) $= 10^7$
9. अर्बुद (10 करोड़) $= 10^8$
10. न्यर्बुद (अब्ज) (1 अरब) $= 10^9$

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4. एकं च दश च शतं च सहस्रमयुत-नियुते तथा प्रयुतम्।
कोट्यर्बुदं च वृन्दं स्थानात् स्थानं दशगुणं स्यात्। आर्यभटीय, गणितपाद 2
5. त्रिशतिका, सूत्र 2-3
6. एकदशशतसहस्रायुत-लक्ष-प्रयुतकोटयः क्रमशः।
अर्बुदमब्जं खर्वनिखर्वमहापद्मशंकवस्तस्मात्।। 2 ।।
जलधिश्चान्त्यं मध्यं परार्धमिति दशगुणोत्तरं सज्ञाः।। 3 ।। लीलावती 2.2-3
7. एवम् एकाद्यष्टादशसंख्यासंज्ञासंमिता इष्टकाः। महीधर, यजु0 17.2

11.	खर्व (10 अरब)	$= 10^{10}$
12.	निखर्व (1 खरब)	$= 10^{11}$
13.	महापद्म (10 खरब) (महासरोज)	$= 10^{12}$
14.	शंकु (1 नील)	$= 10^{13}$
15.	समुद्र (जलधि) (10 नील)	$= 10^{14}$
16.	मध्य (100 नील) (अन्त्य)	$= 10^{15}$
17.	अन्त (अन्त्य) (1 हजार नील) (मध्य)	$= 10^{16}$
18.	परार्ध (10 हजार नील)	$= 10^{17}$

संख्यावाचक शब्दों का आधार 10 संख्या

उपर्युक्त विवरण से ज्ञात होता है कि वैदिक संख्या-पद्धति का आधार 10 अंक रहा है। अतएव दश शत सहस्र आदि संख्याओं को 'दशगुणाः संज्ञाः' या 'दशगुणोत्तरं संज्ञाः' कहा गया है। दश से परार्ध तक की संज्ञाएँ क्रमशः 10 गुनी होती चली गयी है।

दश (10) संख्या के लिए 'ति' प्रत्यय लगाना

दस संख्या के लिए 'ति' प्रत्यय लगाना भाषाविज्ञान की दृष्टि से बहुत महत्वपूर्ण है। 10 के लिए 'ति' का प्रयोग इंग्लिश् में भी होता है और इसका अर्थ होता है- 10 गुना। इंग्लिश् में यह **Ten** (10) का संक्षिप्त रूप '**Ty**' है। यह सिद्ध करता है कि संस्कृत और इंग्लिश् दोनों भाषाओं का उद्गम स्थान एक है। भाषाविज्ञान में इसे भारोपीय भाषा **(Indo-European Language)** कहते हैं, जैसे -

संख्या	संस्कृत	इंग्लिश्
60	षष्टि (षष् + ति)	Sixty (Six+ty)
70	सप्तति (सप्त + ति)	Seventy (Seven+ty)
80	अशीति (अष्ट + ति)	Eighty (Eight+ty)
90	नवति (नव + ति)	Ninety (Nine+ty)

9 अंक के लिए आरोह और अवरोह क्रम

वेदों में 19,29,39 आदि संख्याओं में 9 अंक के लिए आरोह और अवरोह दोनों क्रम अपनाए गए हैं। आरोह क्रम (Ascending order) का अभिप्राय है- संख्या का आगे की ओर बढ़ना। अवरोह क्रम (Descending order) का अभिप्राय है- अगली संख्या देकर पीछे की ओर मुड़ना। हिन्दी में 19,29,39 आदि गिनती में अवरोह-क्रम अपनाया गया है। जैसे 19 के लिए उन्नीस (ऊन-बीस, अर्थात् 20 से 1 कम), 29 के लिए उनतीस (ऊन-तीस, तीस में से एक कम), 69 के लिए उनहत्तर

(ऊन-सत्तर, सत्तर में से एक कम), 79 के लिए उन्नासी (ऊन-अस्सी, अस्सी में से एक कम)। हिन्दी में यह विधि वैदिक विधि से ली गयी है। वेदों में 19, 29 आदि के लिए आरोह और अवरोह क्रम दोनों अपनाए गए है। जैसे-नवदशन् (19), नवषष्टि (69), नवाशीति (89) आदि में आरोह-क्रम अपनाया गया है, अर्थात् 10+9=19, 60+9=69, 80+9=89 । दूसरी ओर 19,29 आदि में 'एक कम' के अर्थ में 'एकोन' 'एकात्र' और केवल 'ऊन' (ऊन=न्यून अर्थात् कम) शब्दों का प्रयोग प्राप्त होता है। एकोनविंशति आदि।

दशम-पद्धति का उल्लेख : अथर्ववेद में एक सूक्त के 11 मंत्रों में दशम-पद्धति का उल्लेख है। इसमें 1 से लेकर एक हजार तक संख्या शब्दों के दशम पद्धति पर नाम दिए हैं।[1] जैसे-1-10, 2-20, 3-30, 4-40, 5-50, 6-60, 7-70, 8-80, 9-90, 10-100, 100-1000। इसी प्रकार का भाव यजुर्वेद में भी है।[2] एकया-दशभिः (1-10), द्वाभ्यां:-विंशती, (2-20), तिसृभिः - त्रिंशता (3-30)। इनमें 1,2,3 आदि का 10,20,30 आदि में साक्षात् संबन्ध दिखाया गया है।

संख्या (Cardinals) और संख्येय (Ordinal numbers) : यजुर्वेद में तीन स्थलों पर प्रत्येक संख्या से संबद्ध संख्येय शब्दों की गणना की गयी है। इसमें 1 से लेकर 48 तक के संख्येय शब्द दिए गए हैं। (1) एक मंत्र में 1 से 12 तक के संख्येय शब्द दिए गये हैं।[3] एक का संख्येय शब्द प्रथम, 2-द्वितीय, 3-तृतीय, 4-चतुर्थ, 5-पंचम, 6-षष्ट, 7-सप्तम, 8-अष्टम, 9-नवम, 10-दशम, 11-एकादश, 12-द्वादश। (2) एक अन्य मंत्र में 13 से 17 तक के संख्येय शब्द दिए हैं।[4] 13-त्रयोदश, 14-चतुर्दश, 15-पंचदश, 16-षोडश, 17-सप्तदश। (3) यजुर्वेद में अन्यत्र 4 मंत्रों में कुछ संख्याओं को छोड़ते हुए 15 से 48 तक के संख्येय शब्द दिए गए हैं।[5] 17 से आगे के संख्येय शब्द ये दिये हैं : 18-अष्टादश, 19-नवदश, 20-विंश, 21-एकविंश, 22-द्वाविंश, 23-त्रयोविंश, 24-चतुर्विंश, 25-पंचविंश, 27-त्रिणव, 31-एकत्रिंश, 33-त्रयस्त्रिंश, 34-चतुस्त्रिंश, 36-षट्त्रिंश, 44-चतुश्चत्त्वारिश, 48-अष्टाचत्वारिंश।

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1. एका च मे दश च मे, द्वे-विंशतिः, तिस्रः-त्रिंशत्.....
   दश-शतम्, शतम्-सहस्रम्। अथर्व0 5.15.1 से 11
2. यजु0 27.33
3. सविता प्रथमेऽहनि, द्वितीये, तृतीये, चतुर्थे..... द्वादशे। यजु0 39.6
4. त्रयोदशम्, चतुर्दशम्, पञ्चदशम् षोडशम्, सप्तदशम्। यजु0 9.34
5. यजु0 14.23 से 26

ऋग्वेद के प्रथम (1.162.4), द्वितीय और तृतीय (1.141.2), अष्टम (2.5.2), नवम (5.27.3), दशम (8.24.3) और शततम (4.26.3) ये प्रयोग मिलते हैं।

आठ परिकर्म (Fundamental operations)

अंकगणित के 8 परिकर्म ये हैं : 1. संकलित (जोड़, जोड़ना, Addition) 2. व्यवकलित (घटाना, Subtraction), 3. गुणन (गुणा करना, Multiplication), 4. भागहार (भाग देना, Division), 5. वर्ग (Square), 6. वर्गमूल (Square-root), 7. घन (Cube), 8. घनमूल (Cuble-root)।

1. संकलित (जोड़, Addition)

दो या अधिक राशियों के जोड़ने को जोड़ या संकलित कहते हैं। वेदों में जोड़ की विधि का वर्णन है। जोड़ के लिए निम्नलिखित शब्द प्रयुक्त होते है: संकलन, योग, मिश्रण, संमेलन, संयोजन, युक्ति, एकीकरण आदि। दो विभिन्न संख्याओं को जोड़ने को संकलन, योग या संकलित कहते है। जोड़ का चिह्न + है। वेदों में जोड़ के लिए इन शब्दों का प्रयोग मिलता है:

(क) 'च' (और) शब्द : अथर्ववेद के एक सूक्त में इसके अनेक उदाहरण है।[1] जैसे - षष्टिः च षट् च (60 + 6 = 66), चत्वारः चत्वारिंशत् च (4 + 40 = 44), त्रयः त्रिंशत् च (3 + 30 = 33), द्वौ च विंशतिः च (2 + 20 = 22)। ऋग्वेद में एक बड़ी संख्या 3339 के लिए च शब्द के द्वारा जोड़ना बताया है। त्रीणि शता, त्री सहस्राणि, त्रिंशत् च, नव च (300 + 3000 + 30 + 9 = 3339)।[2]

(ख) साकम् (साथ) शब्द : ऋग्वेद में 99 के लिए 'नव साकं नवतीः' (9 + 90 = 99)।[3] 360 के लिए 'त्रिंशता साकं षष्टिः' (300 + 60 = 360)

(ग) बिना किसी शब्द के प्रयोग के : किसी शब्द का प्रयोग किए बिना यदि संख्याएँ एक साथ दी जाती है, तो उनका अर्थ 'जोड़' है। जैसे - 11 से 99 तक के शब्द। एकादश (1 + 10 = 11), अष्टाशीति (8 + 80 = 88), नवनवति (9 + 90 = 99)। जोड़ से संबद्ध कुछ शब्द ये हैं : 1. योज्य (जिसमें कोई संख्या जोड़ी जाती है) 2. योजक (जोड़ी जाने वाली संख्या), 3. योग (दोनों संख्याओं का जोड़)।

1. अथर्व0 19.47 3 से 5
2. ऋग्0 3.9.9 ; 10.52.6
3. ऋग्0 4.26.3

2. व्यकलित (घटाना, Subtraction)

किसी राशि में से किसी राशि को घटाने को व्यवकलित या घटाना कहते हैं। घटाने के लिए अन्य शब्द ये है: व्यवकलन, व्युत्कलित, व्युत्कलन, वियोग, शोधन, पातन। जिसमें से घटाया जाता हैं, उसे वियोज्य (या सर्वधन Minuend) और जिसे घटावें, उसे वियोजक (Substrahend) कहते है। जो बचता है, उसे शेष या अन्तर (Remainder) कहते हैं। घटाना का चिह्न (-) है।

वेदों में 'घटाना' के लिए कोई निश्चित शब्द नहीं है। इसके लिए कुछ शब्द दिए गए हैं, जिनसे घटाना अर्थ निकलता है। ये शब्द हैं :

1. **'अवम' (कम)** : अथर्ववेद के एक मंत्र में घटाने के लिए 'अवम' शब्द दिया गया है।[1] इस सूक्त में 11 संख्या से संबद्ध अंक इस प्रकार दिए गए हैं : 99, 88,77,66,55,44,33,22,11 । मंत्र में कहा गया है कि 'एकादशावमा:' अर्थात् प्रत्येक संख्या 11 कम होती गयी है। कम के लिए 'अवम' शब्द है। यह उदाहरण जोड़, घटाना और गुणा तीनों के लिए अत्युत्तम है।जैसे :**(क) जोड़** : नीचे से ऊपर की ओर। 11+11=22, 22+11=33, 33+11=44। इसी प्रकार 99 तक 11 जोड़ते चले जाएँगे। **(ख) घटाना** : ऊपर से नीचे की ओर। 99-11=88, 88-11=77, 77-11=66, 66-11=55, 55-11=44, 44-11=33, 33-11=22, 22-11=11, **(ग) गुणा** : नीचे से ऊपर की ओर गुणा। यह 11 का पहाड़ा 99 तक हो जाता है। 11 X 1 =11, 11 X 2 =22, 11 X 3 =33, 11 X 4 =44, 11 X 5 =55 । इसी प्रकार 11 X 9 =99 तक गुणनफल है।

2. **ऊन, एकोन, एकात्र शब्द (एक कम)** : वेदों में 'एक कम' के लिए इन शब्दों का प्रयोग है। ऊन और न्यून (नि+ऊन) शब्द का एक ही अर्थ है- कम या एक कम। एकोन (एक+ऊन) का अर्थ है- एक कम। एकात्र शब्द एकात्+न=एकात्र है, अर्थात् एक संख्या से कम, एक कम।

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1. नवतिर्नव, अशीतिः अष्टा, सप्त सप्ततिः, षष्टिः च षट्, पंचाशत् पञ्च, चत्वारः चत्वारिंशत् च, त्रयः त्रिंशत् च, द्वौ च विंशतिः, एकादशावमाः। अथर्व0 19.47.3 से 5

3. गुणन (Multiplication)

किसी संख्या को किसी संख्या से गुणा करने को गुणा कहते हैं। जिस संख्या में अन्य संख्या से गुणा किया जाता है, उसे 'गुण्य' (Multiplicand) कहते है। जिस संख्या से गुणा किया जाता है, उसे गुणक (Multiplier) कहते हैं। गुणा करने से जो राशि प्राप्त होती है, उसे 'गुणन फल' (Product of Multiplication) कहते हैं। गुणा के लिए (X) संकेत का प्रयोग किया जाता है। गुणन के अन्य पर्याय हैं - हनन, वध, घात, क्षय आदि।

गुणन के लिए 'हनन' और गुणनफल के लिए 'प्रत्युत्पत्र' शब्दों का भी प्रयोग होता था।

वेदों में गुणन के लिए ये विधियाँ अपनाई गयी है:

(क) बिना प्रत्यय के गुणन-कार्य : कोई शब्द या प्रत्यय लगाए बिना गुणन कार्य होता है। जैसे मैत्रायणी संहिता में उल्लेख है कि 2 वर्षों में 24 और 3 वर्ष में 36 मास होते हैं। 1 वर्ष = 12 मास, 2 वर्ष = 12x2 = 24 मास। 3 वर्ष = 12x3 = 36 मास।[1]

ऋग्वेद में 2 का पहाड़ा 10 तक दिया गया है।[2] जैसे: 2x1 = 2, 2x2 = 4, 2x3 = 6, 2x4 = 8, 2x5 = 10 । एक अन्य मंत्र में 10 का पहाड़ा 20 से लेकर 100 तक किया गया है।[3] जैसे 10x2 = 20, 10x3 = 30, 10x10 = 100 । इसी प्रकार अथर्ववेद में 11 का पहाड़ा 11 से 99 तक दिया है।[4] इसमें उलटी ओर चले हैं। जैसे: 11x9 = 99, 11x8 = 88, 11x3 = 33, 11x2 = 22, 11x1 = 11 । अथर्ववेद में 21 के लिए त्रिषप्त, अर्थात् 3x7=21 दिया है।[5]

(ख) स्-प्रत्यय लगाकर गुणनकार्य : स् प्रत्यय लगाकर द्वि का द्विः और त्रि का त्रिः बनता हैं इसका अर्थ है - दो बार, तीन बार। जैसे ऋग्वेद में 20 के लिए द्विर्दश है, अर्थात् 10x2 = 20 ।[6] काठक संहिता में 33 के लिए 'त्रिः एकादश' प्रयोग है, अर्थात् 60 बार, 11x3 = 33 ।[7] ऋग्वेद में 180 के लिए 'त्रिः षष्टिः' प्रयोग है, अर्थात् 60 तीन बार, 60x3 = 180 ।[8]

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1. त्रयः संवत्सराः, तेषां षट्त्रिंशत् पूर्णमासाः। मैत्रा0 1.10.8
2. द्वाभ्याम्, चतुर्भिः, षड्भिः, दशभिः। ऋग्0 1.18.4
3. विंशत्या, त्रिंशता... नवत्या, शतेन। ऋग्0 2.18.5 और 6
4. नवतिर्नव, अशीतिः अष्टा, ..द्वौ च विंशतिः, एकादश। अ0 19.47.3 से 5
5. ये त्रिषप्ताः0। अ0 1.1.1
6. द्विर्दश। ऋग्0 1.53.9
7. त्रिः एकादशाः त्रयस्त्रिंशाः। काठक0 38.11
8. त्रि षष्टिः। ऋग्0 8.96.8

यजुर्वेद के दो महत्त्वपूर्ण मंत्र

यजुर्वेद में दो अत्यन्त महत्त्वपूर्ण मंत्र हैं। इनमें एक में 1 से 33 तक की विषम संख्याएँ दी हैं।[1] दूसरे में 4 से 48 तक 4 का पहाड़ा दिया है।[2] इन मन्त्रों को एक साथ आमने-सामने पढ़ने पर गणित की अधिकांश क्रियाएँ स्पष्ट दीखती हैं। जोड़, घटाना, गुणा और भाग तो है ही। साथ ही वर्ग और वर्गमूल की प्रक्रिया भी स्पष्ट हो जाती है। ये संख्याएँ इस प्रकार हैं :

(क) विषम संख्याएँ : 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33 ।

(ख) सम संख्याएँ : 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48 ।

1. जोड़ : (क) की दो संख्याओं को जोड़ते जाइये, (ख) भाग में उनके उत्तर मिलते जायेंगे। मंत्र में 3, 5, 7 आदि शब्द दो बार पढ़े गए हैं। अत: ये दोनों ओर कार्य करते हैं। जैसे 1+3 = 4 । 3+5 = 8 । 5+7 = 12 । 7+9 = 16 । 9+11 = 20 । 11+13 = 24 । 13+15 = 28 आदि।

2. घटाना : ठीक इसका उलटा करने पर घटाने की प्रक्रिया हो जाएगी। (ख) भाग में से (क) भाग की संख्याएँ घटाने पर शेष बचने वाली संख्या मिलती जाएगी। केवल (क) भाग में उलटी ओर से 2 घटावें, उत्तर मिलता जाएगा। इसी प्रकार (ख) भाग में 4 घटाते जाएँ, उत्तर मिलता जाएगा।

(क) जैसे- 4-1 = 3 । 8-3 = 5 । 12-5 = 7 । 16-7 = 9 । 20-9 = 11 । 24-11 = 13।

(ख) क भाग की संख्याओं में से 2 घटाने पर पूर्ववर्ती संख्या उत्तर है। जैसे : 33-2 = 31 । 31-2 = 29 ।

(ग) ख भाग की संख्याओं में से उलटी ओर से 4 घटाने पर पूर्ववर्ती संख्या उत्तर है। जैसे : 48-4 = 44 । 44-4 = 44 । 40-4 = 36 ।

3. गुणा करना : ख भाग में 4 का पहाड़ा है। क्रमशः 1, 2, 3 का गुणा करते जाइएगा, उत्तर प्राप्त होता जाएगा। जैसे- 4x1 = 4 । 4x2 = 8 । 4x3 = 12 । 4x4 = 16 । 4x10 = 40। 4x11 = 44 । 4x12 = 48 ।

1. एका च मे तिस्रश्च मे, पञ्च, सप्त, नव, ... त्रयस्त्रिंशत्0। यजु0 18.24
2. चतस्र: अष्टौ, द्वादश, षोडश, विंशति: .... अष्टाचत्वारिंशत्। यजु0 18.25

4. भाग देना : ख भाग में ही क्रमशः 1, 2, 3, 4 का भाग देने पर 4 उत्तर आता जायेगा। जैसे:- $4 \div 1 = 4$ । $8 \div 2 = 4$ । $12 \div 3 = 4$ । $16 \div 4 = 4$ ।

(ख) ख भाग वाली संख्याओं में एक-एक संख्या छोड़ते जायें और 4 का भाग दें तो भाग क वाली संख्यायें क्रमशः उत्तर होंगी। जैसे $4 \div 4 = 1$ । $12 \div 4 = 3$ । $20 \div 4 = 5$ । $28 \div 4 = 7$ । $36 \div 4 = 9$ ।

इन मंत्रों से वर्ग और वर्गमूल निकालने की विधि का आगे वर्णन किया जायेगा।

4. भाग, भागहार (Division)

प्राचीन ग्रन्थों में भाग के लिए भागहार शब्द का प्रयोग हुआ। इसके अन्य पर्याय शब्द हैं- भाजन, हरण, छेदन आदि। जिस संख्या को किसी अन्य संख्या से भाग दिया जाता है, उसे भाज्य या हार्य (Dividend) कहते हैं। भाग देने पर जो उत्तर आता है, उसे लब्धि या लब्ध (Quotient) कहते हैं।

वेदों में 'भाग' शब्द का प्रयोग भाग या हिस्सा (Share) अर्थ में अनेक मंत्रों में हुआ है। ऋग्वेद में भाग का हिस्सा अर्थ में प्रयोग है, 'इन्द्र ने मुझे मेरा हिस्सा दिया'।[1] अथर्ववेद में 'द्विभागधनम्'।[2] इसका अर्थ है- 'बड़े भाई को पूरी सम्पत्ति का दो हिस्सा धन मिला'। अन्य मंत्रों में भी भागः, भागम्, भागस्य, भागे आदि प्रयोग मिलते हैं।[3]

5. भिन्न-परिक्रम (Fractions)

भिन्न या बटा के लिए वेदों में कुछ शब्द मिलते हैं। यजुर्वेद में चतुर्थांश अर्थात् 1/4 के लिए 'पाद' शब्द का प्रयोग हुआ है। मंत्र का कथन है कि परमात्मा का त्रिपाद् अर्थात् 3/4 अंश संसार के बाहर है ओरं एक पाद अर्थात् 1/4 अंश यह संसार है।[4]

मैत्रायणी संहिता में कुछ भिन्नों के लिए ये पारिभाषिक नाम दिए गये हैं : कला 1/16, कुष्ठा 1/12, शफ 1/8, पाद या पद 1/4 ।[5]

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1. मह्यं दीधरो भागम्०। ऋग्० 8.100.1
2. द्विभागधनम् आदाय०। अथर्व० 12.2.35
3. अथर्व० 2.24.1 । 6.84.2 । 8.1.1 । 2067.5
4. त्रिपादूर्ध्व उदैत् पुरुषः पादोऽस्येहाभवत् पुनः। यजु० 31.4
5. कलया ते क्रीणानि; कुष्ठया, शफेन, पदा। मैत्रा० 3.7.7

वेदों में भाग या हिस्से (Parts) के अर्थ में अंश और भाग शब्दों का प्रयोग हुआ है।[1]

आधा अर्थात् 1/2 के लिए अर्ध और नेम शब्दों का प्रयोग हुआ है।[2] एक-तिहाई अर्थात् 1/3 के लिए त्रेधा, त्रिधा शब्द मिलते है।[3] इसी प्रकार डेढ़ (1+1/2) के लिए 'अध्यर्ध' शब्द है।[4] वेदों में समस्त पदों में 'अवि' शब्द का अर्थ है- 6 मास या आधा वर्ष। अत: यजुर्वेद में डेढ़ वर्ष के लिए त्र्यवि शब्द और ढ़ाई वर्ष के लिए पंचावि शब्द तथा साढ़े तीन के लिए तुर्यवाट् (तुर्यवाह्) शब्द हैं।[5]

इसी प्रकार 1/7 और 1/10 के लिए ये प्रयोग है- सप्तनाम् एकम्, दशानाम् एकम्।[6]

ऋग्वेद आदि में सौवें और सौवें हिस्से (1/100) के लए 'शततम्' शब्द का प्रयोग हुआ है।[7] एक-हजारवें हिस्से (1/1000) के लिए 'सहस्रतम' शब्द का प्रयोग तैत्तिरीय संहिता और तांड्य महाब्राह्मण में मिलता है।[8] सहस्रधा (हजारवां हिस्सा) शब्द 1/1000 हिस्से का भाव प्रकट करता है।9

शुल्ब सूत्रों में 'बटा' के अर्थ में 'भाग' शब्द का प्रयोग मिलता है। जैसे, 1/5 के लिए 'पंचम भाग' शब्द, 1/10 के लिए 'दशमभाग' शब्द। साढ़े तीन, साढ़े चार आदि में साढ़े (सार्ध) के लिए अर्ध शब्द और अगली संख्या दी जाती है। जैसे 3+1/2 को अर्धचतुर्थ, 4+1/2 को अर्धपंचम, 49+1/2 को अर्धापंचाशत्।[10]

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1. अंश:। ऋग्0 7.32.12। भागम्। ऋग्0 8.100.1
2. नेम:, अर्ध:। ऋग् 10.27.18
3. त्रेधा त्रयाणि। तैत्ति0 सं0 4.2.2.1 । त्रिधा, ऋग्0 2.3.10
4. अध्यर्धेड। तांड्य महाब्राह्मण 10.12.4
5. त्र्यवि:, पंचावि:, तुर्यवाट्। यजु0 18.26
6. ऋग्0 10.5.6। दशानाम् एकम्। ऋग्0 10.27.16
7. शततमाविवेषी:। ऋग्0 7.19.5
8. सहस्रतम्या, तै0 सं0 7.1.6.1। सहस्रतमी, तां0 महा0 21.1.3
9. सहस्रधा। ऋग्0 10.114.8। अ0 10.7.9
10. B. Datta, The Science of Sulba.PP.212-216

वर्ग, वर्गमूल (Square, Square-root)

वेदों में स्पष्ट रूप से वर्ग (Square), वर्गमूल (Square-root), घन (Cube) और घनमूल (Cube-root) का उल्लेख नहीं है। जैसा कि पहले उल्लेख किया जा चुका है कि वर्ग और वर्गमूल का केवल संकेत है। घन और घनमूल का कोई प्रत्यक्ष या अप्रत्यक्ष संकेत नहीं मिलता है। आर्यभट, भास्कर आदि के ग्रन्थों में वर्ग के लिए 'कृति' शब्द का प्रयोग है।

यजुर्वेद में दो स्थानों पर विषम संख्याओं का उल्लेख है। दोनों स्थानों पर 1 से 33 तक की विषम संख्याएँ दी गयी है।[1] मंत्र 18.24 में 1 से 33 तक विषम संख्याएँ ये दी गयी है : 1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33। इस प्रकार ये 17 अंक हैं। इनमे 1 से 17 तक का वर्ग और वर्गमूल निकाला जा सकता है।

वर्ग निकालना : जिस संख्या का वर्ग निकालना हो, उसके लिए उतनी ही विषम संख्याएँ लेनी होंगी। जैसे 1 के लिए केवल 1 विषम संख्या, 2 के लिए 2 विषम संख्याएँ, 3 के लिए 3 विषम संख्याएँ, 10 के लिए 10 विषम संख्याएँ। जैसे : 1 से 10 तक के वर्ग के लिए 10 तक की विषम संख्याएँ लेकर उन्हें जोड़ लें। वही वर्ग की संख्या होगी।

(1) $1+0=1$

(2) $1+3=4$ (2 का वर्ग) 2^2

(3) $1+3+5=9$ (3 का वर्ग) 3^2

(4) $1+3+5+7=16$ (4 का वर्ग) 4^2

(5) $1+3+5+7+9=25$ (5 का वर्ग) 5^2

(6) $1+3+5+7+9+11=36$ (6 का वर्ग) 6^2

(7) $1+3+5+7+9+11+13=49$ (7 का वर्ग) 7^2

(8) $1+3+5+7+9+11+13+15=64$ (8 का वर्ग) 8^2

(9) $1+3+5+7+9+11+13+15+17=81$ (9 का वर्ग) 9^2

(10) $1+3+5+7+9+11+13+15+17+19=100$ (10 का वर्ग) 10^2

इसी प्रकार 1 से 33 तक 17 विषम संख्याओं का जोड़ 289 होगा और यह 17 संख्या का वर्ग है। श्रीधर, महावीर, भास्कर द्वितीय आदि ने भी इस विधि का उल्लेख किया है।

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1. यजु0 14.28 – 31। यजु0 18.24

वर्गमूल निकालना : वर्ग निकालने में 1,3,5 आदि विषम संख्याओं का जोड़ लेते हैं, किन्तु वर्गमूल निकालने में उन विषम संख्याओं का जोड़ नहीं लेना है, केवल इतना गिनता है कि कितनी विषम संख्या ली गयी है। इस प्रकार वर्ग का वर्गमूल निकलता जाएगा। 1 से 33 तक विषम संख्याओं में केवल 17 वर्गमूल होते हैं। जैसे-

वर्ग

तीन का वर्ग 1+3+5=9

पाँच का वर्ग 1+3+5+7+9=25

छह का वर्ग 1+3+5+7+9+11=36

वर्गमूल

9 का वर्गमूल 3, इसमें 1,3,5 =3 संख्याएँ हैं।

25 का वर्गमूल 5, इसमें 1,3,5,7,9 =5 संख्याएँ।

36 का वर्गमूल 6, इसमें 1,3,5,7,9,11 = 6 संख्याएँ।

इसी प्रकार 17 तक के वर्ग के 17 वर्गमूल निकलते जायेंगे। घन और घनमूल का कोई संकेत प्राप्य नहीं हैं।

शून्य (Zero) का महत्त्व

शून्य के लिए प्रयुक्त शब्द : शून्य के लिए इन शब्दों का प्रयोग मिलता है : ख, गगन, अम्बर, आकाश, अन्तरिक्ष, अनन्त, तुच्छ्य, रिक्त, वशी, वशिक, नभ, पूर्ण।

शून्य और शून शब्द : वेदों में शून्य और शून दोनों शब्दों का प्रयोग रिक्त, खाली, अभाव, रिक्ता (Empty, Emptiness) अर्थ में हुआ है।

मा शूने भूम । ऋग्0 1.105.3 (हम कभी अभावग्रस्त न हों)

शून्यैषी निर्ऋते। अ0 14.2.99 (दरिद्रता अभाव करती है)

अशून्योपस्था। छान्दोग्य ब्रा0 1.1.11 (गोदभरी, पुत्रादि से युक्त)

शून्य और जीरो (Zero) का सम्बन्ध : भाषाविज्ञान की दृष्टि से शून्य, जीरो और साइफर (Cipher, Cypher) ये शब्द परस्पर संबद्ध हैं। शून्य का दो प्रकार से विकास हुआ। शून्य शब्द का अरबी में अनुवाद हुआ-सिफ्र। यह सिफ्र दो मार्गो से होता हुआ यूरोप पहुँचा और वहाँ साइफर और जीरों हुआ।

	संस्कृत	अरबो	स्पेनिश	फ्रेंच	इंग्लिश्
1.	शून्य	सिफ्र	सिप्रा	सिफ्रे	साइफर (Cipher)
2.	शून्य	सिफ्र	लेटिन-जैफ्रम	ज़ीरो	ज़ीरो (Zero)

अरबी का सिफ्र पुरानी फ्रेंच में सिफ्रे हुआ और इंग्लिश् में साइफर। वही नई फ्रेंच और इंग्लिश् में ज़ीरो हुआ।

शून्य क्या है ?

शून्य वस्तुत: विश्व के लिए एक पहेली है। इसका तात्त्विक विवेचन आज तक पूर्ण नहीं हुआ है। यह सृष्टि का आदि और अन्त है। इससे ही सृष्टि का प्रारम्भ होता है और इसमें ही लय होता है। इसको दार्शनिकों और वैज्ञानिकों ने अलग-अलग नाम दिए हैं। शून्य के लिए ही वेदों में 'ख' शब्द का प्रयोग हुआ है। 'ख' के अनेक अर्थ वेदों में दिए गए हैं। ख का अर्थ है - आकाश, इन्द्रिय, रिक्त स्थान, छिद्र, द्वार, अन्तरिक्ष, स्वर्ग या देवलोक। जैसे -

1. **खे रथस्य।** ऋग्0 8.91.7 (रथ के छिद्र में)
2. **क: सप्त खानि वि ततर्द शीर्षणि** **(अ0 10.2.6)** **(किसने 7 इन्द्रियाँ बनाई)**
3. **अंग्धि खम्।** ऋग्0 10.156.3 (आकाश को जल से सिक्त करो)
4. **विषाहि....गृणते... खम्।** ऋग्0 4.11.2 (हे अग्नि, भक्त को स्वर्ग दो)
5. **ओं खं ब्रह्म।** यजु0 40.17 (ब्रह्म आकाशवत् शून्य है)

शून्य का मान : शून्य के विषय में एक सुन्दर श्लोक मिलता है, जिसका भाव है अंक के साथ दाहिनी ओर शून्य रखने से उस अंक का मान दस गुना अधिक हो जाता है। अंकों को पढ़ने में दाहिने से बाईं ओर जाना होता है। अत: 'अंकानां वामतो गति:' कहा गया है।

अंकेषु शून्यविन्यासाद्, वृद्धिः स्यात् तु दशाधिका।
तस्माद् ज्ञेया विशेषण, अंकानां वामतो गतिः।।

समयोचित - पद्यरत्नमालिका।

दशमलव - स्थानमान - पद्धति : यह पद्धति भारत का सर्वोत्कृष्ट आविष्कार है। इस पद्धति में 1 से 9 तक के अंक हैं तथा दसवाँ शून्य है। इसमें केवल 10 चिह्न हैं, जिनके स्थानिक मानों को दशम पद्धति पर मान देकर सभी संख्याओं को व्यक्त किया जा सकता है। यही पद्धति विश्व के समस्त सभ्य देशों में प्रयुक्त हो रही है। शून्य के आविष्कार के कारण दश, शत, सहस्र आदि संख्याओं को व्यक्त करना संसार के सबसे बड़े आविष्कारों में एक गिना गया है।

गणित के मूर्धन्य विद्वान् प्रो० हाल्सटेड (G.B. Halsted) ने शून्य की महत्ता का वर्णन करते हुए कहा है - 'शून्य के आविष्कार के महत्त्व की प्रशंसा कभी भी अतिशयोक्तिपूर्ण नहीं कही जा सकती है। निरर्थक शून्य को केवल स्थान, संज्ञा, आकृति एवं संकेत ही नहीं, अपितु एक उपयोगी शक्ति प्रदान करना हिन्दू जाति की एक विशेषता है। यह निर्वाण के विद्युत् - शक्ति में परिवर्तित करने के तुल्य है। गणित संबंधी कोई भी एक अविष्कार ज्ञान एवं शक्ति को आगे बढ़ाने में इतना प्रबल सिद्ध नहीं हुआ है।'[1]

हाल्सटेड का यह कथन सर्वथा सत्य है कि दशमलव - स्थानमान - पद्धति के आविष्कार ने शून्य को इतना अधिक महत्वपूर्ण बना दिया है कि यह निरर्थक समझा जाने वाला शून्य बहुमूल्य रत्न बन गया है।

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1. "The Importance of the creation of Zero-mark can never be exaggerated. This giving to airy nothing, not merely a local habitaiton and a name, a picture, a symbol, but helpful power, is the characteristic of the Hindurace whence it sprang. It is like coining the Nirvana into dynamos, No single mathematical creation has been more potent for the general on-go of intelligence and power."

-G.B. Halsted-On the foundation and technique of Arithmetic, 1912, P.20

वेदों में अतएव अनन्त[1] अपरिमित[2], असंख्यात[3], असख्येय[4] आदि शब्द शून्य-स्थान के महत्त्व के बोध के लिए प्रयुक्त हुए हैं। कहीं पर ये शब्द ब्रह्म, शिव आदि के सूचक हैं और कहीं विभिन्न शक्तियों के लिए हैं। ऋग्वेद के एक मंत्र में 'दशान्तरुष्यादतिरोचमानम्' में दश (10) के महत्त्व का वर्णन करते हुए कहा गया है कि इससे इसकी शक्ति गुप्त रूप से बहुत बढ़ती जाती है।[5] 'दशान्तरुष्य' का अर्थ है - दस की संख्या के कारण गुप्तरूप से शक्ति का बढ़ जाना। अतएव सायण ने अर्थ किया है -

अन्तरुष्यं गूढम् आवासस्थानम्। तच्च स्थानं दशसंख्योपेतम्।[6]

परार्ध और अवरार्ध : यजुर्वेद में एक से लेकर परार्ध तक की संख्याओं का उल्लेख है।[7] परार्ध संख्या 18वां स्थान है। मंत्र में परार्ध शब्द बहुत महत्त्वपूर्ण है। परार्ध का अर्थ है - पर अर्थात् उत्कर्ष की ओर, अर्ध - आधा भाग। इसका अभिप्राय यह है कि धनात्मक संख्या एक (+1) से लेकर 18वें स्थान तक बढ़ते चले जाएँ तो प्रत्येक संख्या 10 गुनी होती चली जाएगी। अतएव इनको 'दशगुणोत्तर संज्ञा' कहा गया है।[8] परार्ध का अभिप्राय यह है कि 18 स्थान तक धनात्मक संख्याओं के जो ये नाम दिए गये हैं, वह पूरी संख्या का आधा भाग हैं। इसका आधा भाग 'ऋणात्मक संख्याएँ' हैं। इनको वेद में 'अवरार्ध' अर्थात् 'ऋणात्मक आधा भाग' कहा गया है। शतपथ ब्राह्मण में 'अवरार्धतः' और काण्व संहिता में 'अवरार्धः' का प्रयोग हुआ है।[9] इसका अर्थ यह निकलता है कि जिस प्रकार 18 स्थान तक 'धनात्मक संख्याएँ' हैं, उसी प्रकार 18 स्थान तक 'ऋणात्मक संख्याएँ' होंगी और उनको शत, सहस्र आदि के आधार पर शतांश, सहस्रांश, लक्षांश (100वाँ, 1000वाँ आदि) कहा जाएगा। यह गुणा (गुना) के विरुद्ध भाग (हिस्सा) अर्थ बताएगा। परार्ध और अवरार्ध शब्द दशमलव (शून्य) से पूर्व और बाद का अर्थ बताते हैं। यजुर्वेद में 'अतिदीर्घ' और 'अतिह्रस्व' दो शब्द आए हैं।[10] अतिदीर्घ सूचित करता है कि बहुत बड़ी संख्या धनात्मक वृद्धि करते हुए 'परार्ध' तक जाएगी और 'अतिह्रस्व' बताता है कि बहुत छोटी संख्या ऋणात्मक रूप से घटते हुए 'अवरार्ध' 10^{-17} तक जाएगी।

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1. अनन्तः। ऋग्0 1.113.3
2. अपरिमितो यज्ञः। अ0 9.5.21
3. असंख्याता सहस्राणि ये रुद्राः। यजु0 16.54
4. शतं सहस्रम् अयुतं न्यर्बुदम् असंख्येयम्। अ0 10.8.24
5. ऋग्0 10.51.3
6. सायण, ऋग्0 10.51.3
7. एका च ... अन्तश्च परार्धश्च। यजु0 17.2
8. दशगुणोत्तरं संज्ञाः। लीलावती, श्लोक 3
9. अवरार्धतः। शत0 9.1.2.16 । अवरार्धः। काण्व0 4.1.3.1
10. अतिदीर्घं चातिह्रस्वम्0। यजु0 30.22

शून्य का अभिप्राय : शून्य का अभिप्राय 'अभाव' या 'नहीं' समझना बहुत बड़ी भूल है। शून्य का अभिप्राय पाणिनि के एक सूत्र 'अदर्शनं लोपः' (अष्टा0 1.1.60) से स्पष्ट होता है। व्याकरण में 'लोप' शब्द का अर्थ होता है-किसी वर्ण आदि का हट जाना या अदृश्य होना। पाणिनि ने स्पष्ट किया है कि लोप होने का अभिप्राय है-उस वर्ण आदि का अदर्शन (अदृश्य) हो जाना, न कि उसका अभाव। इसी प्रकार 'शून्य' का अर्थ है-वहाँ पर कोई संख्या अदृश्य रूप में विद्यमान है, जिसको हम एक-दो आदि अंकों से नहीं बता सकते हैं। यदि वस्तुतः शून्य का अर्थ 'अभाव' हो तो 10, 100, 1001 आदि संख्याएँ बन ही नहीं सकती हैं। हम 10, 100 और 1000 को एक ही कहेंगे, दस, सौ, एक हजार नहीं, क्योंकि 1 संख्या के आगे एक, दो या तीन शून्यों का कोई अर्थ नहीं है, वे हैं ही नहीं। वास्तविकता यह है कि शून्य, कोई विशेष अंक न होने पर भी, अपना स्थान बनाए हुए है और वह जिस स्थान पर है, उसका स्थानमान बताता है। नहीं तो 101 और 1001 को 11 पढ़ा जायेगा।

शून्य और अनन्त : यजुर्वेद (40.17) में 'ओं खं ब्रह्म' कहकर शून्य को अनन्त और अपरिमेय बताया गया है। किसी भी संख्या में शून्य को जोड़ें, घटावें या गुणा करें तो उसका मान वही रहता है। उसमें कोई परिवर्तन नहीं होता है। भास्कराचार्य द्वितीय (1150 ई0) ने सर्वप्रथम यह स्पष्ट किया है कि किसी भी संख्या को शून्य से भाग देने पर 'अनन्त' संख्या आती है। इस अनन्त राशि को ही 'खहर' कहा जाता है।

वधादौ वियत् खस्य खं खेन घाते।
खहारो भवेत् खेन भक्तश्च राशिः।
अयमनन्तो राशिः खहर इत्युच्यते। बीजगणित श्लोक 3

भास्कर ने इस 'खहर' राशि की तुलना विष्णु (ब्रह्मा, अच्युत, ईश्वर) से की है। उसका कथन है :

अस्मिन् विकारः खहरे न राशौ-
अपि प्रविष्टेष्वपि निःसृतेषु।
बहुष्वपि स्याद् लय-सृष्टि-कालेऽ-
नन्तेऽच्युते भूतगणेषु यद्वत्। बीज0 श्लोक 4

अर्थात् प्रलय और सृष्टि के समय अनन्त अच्युत (विष्णु) में समस्त प्राणियों के लीन एवं निर्गत होने पर जैसे उसमें कोई विकार नहीं होता, उसी प्रकार इस 'खहर' राशि में किसी राशि को घटाने, जोड़ने आदि से कोई विकार (अन्तर, परिवर्तन) नहीं होता है। यही भाव निम्न श्लोक में भी मिलता है कि पूर्ण में से पूर्ण निकाल लेने पर भी पूर्ण ही बचता है, अर्थात् शून्य में से शून्य को निकाल लेने पर भी शून्य (पूर्ण) शेष बचता है। शून्य आकाश के तुल्य एक पूर्ण संख्या है। उसमें से घटाने आदि से कोई अन्तर नहीं पड़ता।

पूर्णमदः पूर्णमिदं, पूर्णात् पूर्णमुदच्यते।
पूर्णस्य पूर्णमादाय, पूर्णमेवावशिष्यते।। उपनिषदों में शान्तिपाठ

वेद में दशमलव पद्धति का संकेत : ऋग्वेद के दो मंत्रों में दशमलव पद्धति का संकेत मिलता है। इनमें यह भी संकेत मिलता है कि दशमलव पद्धति कितने प्रकार से काम कर सकती है और इसके क्या लाभ हैं।

दशावनिभ्यो दशकक्ष्येभ्यो
दशयोक्त्रेभ्यो दशयोजनेभ्यः।
दशाभीशुभ्यो अर्चताजरेभ्यः
दश धुरो दश युक्ता वहद्भ्यः।।
ते अद्रयो दशयन्त्रास आशव-
स्तेषामाधानं पर्येति हर्यतम्।। ऋग्0 10.94.7 और 8

इन दो मंत्रों में दशमलव पद्धति की इतनी विस्तृत व्याख्या की गई है कि उसका वर्णन करना संभव नहीं है। जिन शब्दों का मंत्रों में प्रयोग हुआ है, वे गूढ़ शब्द है। जो विशेषताएँ प्रकट होती हैं, वे संक्षेप में ये हैं :

1. **दश अवनि** : दशमलव पद्धति के प्रयोग के 10 क्षेत्र हैं। 1 से 10 तक के अंक 10 क्षेत्रों में प्रयुक्त होकर 10x10=100 क्षेत्र बनाते हैं।
2. **दश कक्ष्य** : इनकी 10 कक्षाएँ हैं। जोड़-घटाना, गुणा, भाग, वर्ग, वर्गमूल, घन, घनमूल आदि।
3. **दश योक्त्र** : इनको 10 प्रकार से प्रयुक्त कर सकते हैं। रेखागणित आदि में 10 प्रकार के कोण आदि बनाना।
4. **दश योजन** : इनको 10 प्रकार से जोड़-घटाना आदि में लगा सकते हैं।
5. **दश अभीशु** : ये 10 प्रकार के अभीषु (Rein, Ray, Peg) शंकु हैं। इनको जिधर चाहें, उधर मोड़ सकते हैं।
6. **दश धुरा** : ये 10 प्रकार की धुरा (Yokes) है। इन्हें 10 प्रकार से प्रयोग में ला सकते हैं।
7. **दश यन्त्र** : ये 10 अंक दश यंत्र (मशीन) का काम करते हैं। इन दस अंकों को जहाँ चाहें, गणित में मशीन के तुल्य प्रयोग कर सकते हैं। इनमें 10 प्रकार से नियंत्रण (control) की क्षमता है। संख्या को चाहे धनात्मक, ऋणात्मक, गुणात्मक या विभाजक जो बनाना चाहे, बना सकते हैं।

8. अद्रय : अद्रि का अर्थ- पर्वत, वज्र आदि है। इन अंकों से बड़ी से बड़ी संख्या को वज्र की तरह तोड़ सकते हैं।

9. आशव : आशु का अर्थ है - शीघ्र, तीव्र। दशमलव पद्धति अत्यन्त सरल और शीघ्र प्रभावकारी है। दशमलव पद्धति से कठिन से कठिन प्रश्न सरलता से, शीघ्र हल हो सकते हैं।

10. आधानं पर्येति हर्यतम् : इस दशमलव पद्धति का प्रयोग अति रमणीय और सुन्दर है। इसका प्रयोग सर्वतोगामी है।

उपर्युक्त वर्णन से ज्ञात होता है कि दशमलव पद्धति सर्वश्रेष्ठ पद्धति है। इसके द्वारा सभी प्रकार के प्रश्नों के उत्तर सरलता और शीघ्रता से प्राप्त किये जा सकते हैं।

बीजगणित (Algebra)

बीजगणित का अभिप्राय उस गणित से है, जिसमें अंकों की सहायता के बिना संकेताक्षरों या वर्णों से गणित की क्रिया की जाती है। इस दृष्टि से वेदों में कोई सामग्री प्राप्त नहीं होती है। कतिपय संकेताक्षर मिलते हैं, परन्तु उसका प्रयोग गणितीय दृष्टि से नहीं हुआ है। जैसे-

ख = शून्य या आकाश। (खं ब्रह्म, यजु० 40.17)

क = प्रजापति या ईश्वर। (प्रजापतिर्वै कः, ऐत० ब्रा० 2.38)

परकालीन साहित्य में इस प्रकार के संकेताक्षरों का बहुत प्रयोग हुआ है। लीलावती आदि में ऐसे संकेताक्षर अनेक हैं। जैसे- भ(27), नन्द (9), अग्नि (3), ख (0), बाण (5), सूर्य (12), शैल (7)। अंक दाएँ से बाईं ओर पढ़े जाते हैं, अतः भनन्दाग्नि (3927), ख-बाण-सूर्य (1250) होते हैं। लीलावती, श्लोक-98 ।

ज्यामिति या रेखागणित (Gemoetry)

वेदों में रेखागणित-संबंधी सामग्री सूत्ररूप में मिलती है। इसका विस्तृत विवरण शुल्बसूत्रों में मिलता है। वेदों में जो सामग्री मिलती है, उसकी संक्षिप्त रूपरेखा दी जा रही है।

ऋग्वेद के कुछ मंत्रों में रेखागणित के कुछ पारिभाषिक शब्द मिलते हैं। यज्ञवेदी के संबंध में प्रश्न किया गया है कि इसका क्या नाम था, इसकी क्या रूपरेखा थी, इसकी क्या परिधि थी, आदि।

कासीत् प्रमा प्रतिमा किं निदानम्, आज्यं किमासीत् परिधिः क आसीत्।
छन्दः किमासीत् प्रउगं किमुक्थं यद् देवा देवमयजन्त विश्वे।।

ऋग्० 10.130.3

इस मंत्र में रेखागणित से संबद्ध ये शब्द हैं :

1. प्रमा - नाप, परिमाण (Measurement)
2. प्रतिमा - नक्शा, रूपरेखा (Outline)
3. निदानम् - कारण, मूल सिद्धान्त (Basic Principles)
4. परिधि - घेरा (Circumference)
5. छन्द - नापने का साधन, रज्जू आदि। इसी को शुल्ब कहा गया है।
6. प्रउग - शुल्बसूत्रों में समद्विबाहु त्रिभुज (Isosceles triangle) के लिए इस शब्द का प्रयोग है।

ऋग्वेद के एक मंत्र में वृत्त (Circle) के बारे में आवश्यक विवरण मिलता है।

चतुर्भिः साकं नवतिं च नामभिः
चक्रं न वृत्तं व्यतींरवीविपत्। ऋग्0 1.155.6

मंत्र का कथन है कि एक वृत्त में 4 X 90 = 360 अंश होते हैं। ऐसे कालचक्र को विष्णु घुमाता है। इस मंत्र में स्पष्ट संकेत है कि एक वृत्त में 90 अंश के 4 खंड (त्रिज्या, Radius) होते हैं।

ऋग्वेद और अथर्ववेद में एक मंत्र मिलता है, जिसमें रेखागणित से संबद्ध कई महत्त्वपूर्ण तथ्य प्राप्त होते हैं।

द्वादश प्रधयश्चक्रमेकं त्रीणि नभ्यानि क उ तच्चिकेत।

तस्मिन् साकं त्रिशता न शंकवो अर्पिताः षष्टिर्न चलाचलासः।।

ऋग्0 1.164.48, अथर्व0 10.8.4

इसमें वर्षचक्र का वर्णन करते हुए ये पारिभाषिक शब्द दिये गये हैं। एक चक्र (Circle) है, उसमें 12 प्रधियाँ हैं, अर्थात् 30-30 अंश पर 12 अरे हैं। पूरे चक्र में 120 अंश वाल 3 केन्द्र-बिन्दु है। पूरे चक्र में 360 अंश है।

यजुर्वेद के एक मंत्र में त्रिभुज की रूपरेखा दी गयी है।

तिरश्चीनो विततो रश्मिरेषाम्।
अधः स्विदासीद् उपरि स्विदासीत्।

यजु0 33.74

यह सूर्य की किरणों का वर्णन है। ये तिरछी आती हैं, फिर नीचे फैलती है और फिर ऊपर तिरछी जाती हैं। इस प्रकार त्रिभुज की तीन भुजाएँ होती हैं। नीचे एक रेखा और दोनों ओर दो तिरछी रेखाएँ। इस प्रकार त्रिकोण की अनेक आकृतियाँ बनाई जा सकती हैं।

यज्ञवेदी और रेखागणित

ऋग्वेद[1] और तैत्तिरीय संहिता[2] में अग्नि को त्रिषधस्थ अर्थात् तीन स्थानों पर विराजमान कहा गया है। तीन प्रकार की अग्नियाँ हैं- 1. गार्हपत्य, 2. आहवनीय, 3. दक्षिण। शतपथ ब्राह्मण में इनके आकार का वर्णन है।[3] गार्हपत्य अग्नि की वेदी मंडलाकार (Circle), आहवनीय की चतुर्भुज (Square) और दक्षिणाग्नि की अर्धवृत्ताकार या अर्धचन्द्राकार (Semi-Circle) होती है। इसके लिए यह भी विधान है कि इनका क्षेत्रफल भी बराबर हो और यह एक वर्ग व्याम हो। (एक व्याम = 96 अंगुल या 4 हाथ अर्थात् लगभग 6 फीट)।[4]

इसके लिए यह आवश्यक है कि वेदी बनाने वाले को रेखागणित का ज्ञान हो, जिसे वह वृत्त को चतुर्भुज में बदल सके और उसको अर्धवृत्ताकार बना सके।

शतपथ ब्राह्मण में कुछ वेदियों के निर्माण की विधि दी गई है।[5] विभिन्न प्रकार की वेदियों के निर्माण की विस्तृत विधि तथा वृत्त को चतुष्कोण, चतुष्कोण को त्रिकोण एवं चतुष्कोण को वृत्त में परिवर्तन आदि की पूरी विधि रेखागणित के अनुसार समझाई गई है। सारे शुल्बसूत्र ग्रन्थ एक प्रकार से रेखागणितीय ग्रंथ है। इनमें सामान्य से कठिनतम वेदियों के निर्माण की पूरी विधि वर्णित है।

रेखागणित की दृष्टि से विशेष महत्वपूर्ण ये 4 शुल्बसूत्र हैं : 1. बौधायन शुल्बसूत्र, 2. आपस्तम्ब शुल्बसूत्र, 3. कात्यायन शुल्बसूत्र, 4. मानव शुल्बसूत्र।

विभिन्न प्रकार की वेदियों के निर्माण की विधि, उनके आकार-परिवर्तन आदि की विधि के विस्तृत और व्यापक अध्ययन के लिए कुछ उपयोगी ग्रन्थ भी लिखे गए हैं।[6]

डा0 ए0 के0 बाग (A.K. Bag) ने विभिन्न वेदियों के आकार ओर परिमाण की संक्षिप्त रूपरेखा दी है।[7]

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1. अग्ने .... त्रिषधनस्थ। ऋग्0 5.4.8
2. अग्निं नर: त्रिषस्थे समिन्धते। तैत्ति सं0 4.4.4.3
3. गार्हपत्य: परिमण्डलम्0। शत0 7.1.1.37
4. शत0 3.5.1.1 से 6
5. शत0 ब्रा0 कांड 3 और कांड 7
6. विस्तृत अध्ययन के लिए देखें-

(क) डा0 सत्यप्रकाश- Founders of Sciences in Ancient India. P. 603-675

(ख) डा0 सत्यप्रकाश- The Sulba Sutras, भूमिका पृ0 1-73

(ग) डा0 विभूतिभूषण दत्त- The Science of the Sulba

7. A.K. Bag, Mathematics in India.P.106

वेदी का नाम		आकार	परिमाण
	वर्ग 1		
1.	आहवनीय	चतुर्भुज, Square	1 वर्ग व्याम
2.	गार्हपत्य	वृत्त, Circle	1 वर्ग व्याम
3.	दक्षिणाग्नि	अर्धवृत्त, Semi-Circle	1 वर्ग व्याम
	वर्ग 2		
1.	महावेदी या सौमिक वेदी	समद्विबाहु चतुर्भुज, Isosceles Trap.	972 वर्ग पद
2.	सौत्रामणी वेदी	समद्विबाहु चतुर्भुज, Isosceles Trap.	महावेदी का 1/3 (324 वर्ग पद)
3.	पैतृकी वेदी	समद्विबाहु चतुर्भुज, Isosceles Trap.	सौत्रमणी का 1/9
4.	प्राग्वश	आयत, Ractangle	सौत्रमणी का 1/9
	वर्ग 3		
1.	चतुरस्र श्येनचित्	पक्षी का आकार	साढ़े सात वर्ग पुरुष
2.	वक्रपक्ष-व्यस्तपुच्छश्येन	पक्षी का आकार	साढ़े सात वर्ग पुरूष
3.	कंकचित्	पक्षी का आकार	साढ़े सात वर्ग पुरुष
4.	प्रउग	त्रिभुज, Triangle	साढ़े सात वर्ग पुरूष
5.	उभयत: प्रउग	समबाहु चतुर्भुज, Rohmbus	साढ़े सात वर्ग फुट
6.	रथचक्रचित्	वृत्त, Circle	साढ़े सात वर्ग पुरूष
7.	द्रोणचित्	दोने की आकृति, Trough	साढ़े सात वर्ग पुरूष
8.	श्मशानचित्	समद्विबाहु चतु0, Iso.Trap.	साढ़े सात वर्ग पुरुष

महावेदी

समद्विबाहु चतुर्भुज (Isosceles Trapezium)

शतपथ ब्राह्मण में महावेदी का बहुत विस्तार से वर्णन किया गया है। कांड 3 और कांड 10 में इसके निर्माण का वर्णन किया गया है।[1] इस वेदी का संबन्ध सोमयोग से है, अतः इसे

1. शत0 ब्रा0 3.5.1.1 से 6 और कांड 10.2.3.7 से 14

'सौमिकी वेदी' भी कहते हैं। इसकी रचना समद्विबाहु चतुर्भुज (Isosceles Trapezium) होती है। इसका मुख पूर्व की ओर होता है।

पूर्व दिशा चौड़ाई -	24 पग (12 + 12 के 2 भाग)
पश्चिम दिशा चौड़ाई -	24 + 6 = 30 पग (15 + 15 के 2 भाग)
लंबाई -	36 पग (पद)

1 पद प्राय: 2 फीट का होता है। उसी हिसाब से इसकी लंबाई चौड़ाई समझनी चाहिए। इसमें पूर्व की ओर से पश्चिम की ओर 24 x 24 पग का एक चतुर्भुज बनता है। पश्चिम दिशा में मध्य भाग से उत्तर और दक्षिण की ओर 3 - 3 पग और बढ़ाते हैं। इस प्रकार 6 पग बढ़ जाने से पश्चिम दिशा में चौड़ाई 24 + 6 = 30 पग हो जाती है। उत्तर और दक्षिण दिशा के भाग समान हैं, अत: यह समद्विबाहु चतुर्भुज है। पूर्व और पश्चिम के भाग विषम हैं (24 और 30 पग), अत: पूर्व और पश्चिम की भुजाएँ विषम हैं।

बौधायन शुल्बसूत्र (1.4.3) और आपस्तम्ब शुल्बसूत्र (5.1 से 5) में भी महावेदी का नाप दिया गया है।[2] पद (पैर, Step) के लिए प्रक्रम और विक्रम शब्दों का भी प्रयोग होता है।

शतपथ ब्राह्मण का कथन है कि जो सामान्य सप्तविध वेदी है, उससे यह महावेदी 14 गुनी बड़ी होती है।[3] इस महावेदी के नाम के लिए शुल्ब (रज्जु, रस्सी) का प्रयोग किया जाता है। लंबाई के लिए 36 पग लंबी (रस्सी) ली गयी, पूर्व की चौड़ाई के लिए 24 पग और पश्चिम की चौड़ाई के लिए 30 पग लंबी रस्सी। प्रत्येक वेदी में क़िस आकार की कितनी ईंटें लगती हैं, इसका भी पूर्ण विवरण ब्राह्मण ग्रन्थों में दिया गया है।

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2. त्रिंशत् पदानि प्रक्रमा वा पश्चात् तिरश्ची। षट्त्रिंशत्,
प्राची चतुर्विंशति: पुरस्तात् तिरश्चीति महावेदेर्विज्ञायते। बौधा० 1.4.3

3. यावत्येषा सप्तविधस्य वेदिस्तावतीं चतुर्दश कृत्वा एकशतविधस्य वेदिं विमिमीते।
शत० 10.2.3.7

परिधि, व्यास और ''पाई'' का मान

(Circumference, Diameter, Value of Pie, π)

ऋग्वेद के एक मंत्र में वृत्त की परिधि और व्यास का अनुपात त्रित' शब्द के द्वारा दिया गया है। मंत्र का अर्थ है कि त्रित ने वल राक्षस की परिधि को तोड़ दिया। इसका अभिप्राय है कि वृत्त की परिधि को त्रित अर्थात् व्यास के अनुपात 1/3 ने तोड़ दिया। यह अनुपात वस्तुतः 22/7 है। इस अनुपात को पाई (Pie, π) द्वारा सूचित किया जाता है। इसी प्रकार का भाव अथर्ववेद के एक मंत्र में दिया गया है कि ब्रह्म इस संसार में त्रिभुज के रूप में अपनी आकृति बनाकर रहता है अर्थात् वृत्त (ब्रह्माण्ड) के अन्दर ब्रह्म द्युलोक, अन्तरिक्ष और पृथिवी इन तीन का त्रिकोण बनाकर रहता है।

(क) भिनद् वलस्य परिधीन् इव त्रितः। ऋग्0 1.52.5

(ख) योनिं कृत्वा त्रिभुजं शयानः। अथर्व0 8.9.2

ऋग्वेद के एक मंत्र में यह भी संकेत है कि त्रित अर्थात् परिधि और व्यास का अनुपात 1/3 कहना कुछ त्रुटिपूर्ण है, अतः ऐसा 'त्रित' (त्रिकोण) कुएँ में डूब गया। बृहस्पति (विद्वानों) ने इसका संशोधन किया है और वह 'त्रित' (त्रिकोण) सुरक्षित बाहर आया। इसका अभिप्राय यह है कि स्थूल दृष्टि से यह अनुपात 1/3 हो सकता है, परन्तु इसका शुद्ध रूप 22/7 होना चाहिए।

त्रितः कूपेऽवहितो देवान् हवत ऊतये,
तत् शुश्राव बृहस्पतिः कृण्वन् अंहूरणादुरु। ऋग्0 1.105.17

आर्यभट(476ई0) ने 'आर्यभटीय' के गणितपाद में इससे अधिक सूक्ष्मान दिया है। वृत्त की परिधि और व्यास का यह अनुपात चार दशमलव स्थान तक शुद्ध है। आर्यभट का कथन है कि 104 को 8 से गुणा करो और 62 हजार जोड़ो अर्थात् 62, 832 उस वृत्त की परिधि का आसन्न मान (निकटतम मान) है, जिसका व्यास 20,000 है।

$$\frac{\text{परिधि}}{\text{व्यास}} = \frac{62,832}{20,000} = 3.1416$$

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम्।
अयुतद्वय-विष्कम्भस्यासन्नो वृत्तपरिणाहः।। गणितपाद 10

इस श्लोक में आर्यभट का यह कथन अत्यन्त महत्त्वपर्ण है कि वृत्त की परिधि और व्यास का यह अनुपात 'आसन्नमान' अर्थात् निकटतम् है। इसका अभिप्राय यह है कि 'पाई' एक अपरिमेय संख्या है, इसका ठीक- ठीक मान प्राप्त करना असंभव है। नीलकंठ ने स्पष्ट लिखा है

कि 'पाई' एक अपरिमेय संख्या है। आधुनिक विज्ञान भी आज तक इसका अन्त नहीं पा सका है। अतः इसको अपरिमेय माना जाता है।

भास्कराचार्य द्वितीय (1114 ई0) ने लीलावती में ''पाई'' का स्थूल और सूक्ष्म दोनों मान दिए हैं। भास्कर का श्लोक है:

व्यासे भनन्दाग्नि (3927) -हते विभक्ते
खबाणसूर्यैः (1250) परिधस्तु सूक्ष्मः।
द्वविंशति (22) घ्ने विहृतेऽथ शैलैः (7)
स्थूलोऽथवा स्याद् व्यवहारयोग्यः।। लीलावती, क्षेत्रव्यवहार 98

इसमें ''पाई'' का सूक्ष्ममान इस प्रकार दिया है:

(क) 3927/1250 = 3.1416
(ख) पाई का स्थूल मान : 22/7 = 3.14

भास्कराचार्य ने क्षेत्रव्यवहार के इस प्रकरण में व्यास से परिधि और से व्यास निकालने का सूत्र बताया है।

कर्ण (Hypotenuse) निकालना
पैथागोरस का प्रमेय (Pythagorean Theorem)

बौधायन, आपस्तम्ब और कात्यायन शुल्ब सूत्रों में आयत (Rectangle) के कर्ण (Hypotenuse) के विषय में कहा गया है कि आयत का कर्ण दोनों क्षेत्रफलों को उत्पन्न करता है, जिसे उसकी लम्बाई और चौड़ाई अलग-अलग उत्पन्न करती है।

दीर्घचतुरस्रस्याक्ष्णया रज्जुस्तिर्यङ्मानी पार्श्वमानी च यत् पृथग्भूते कुरुतः, तदुभयं करोतीति क्षेत्रज्ञानम्।

बौधायन शुल्ब0 1.48; कात्यायन शुल्ब0 2.11; आप0 शुल्ब 1.7

अर्थात् किसी आयत के कर्ण पर खींचा गया वर्ग, क्षेत्रफल में उन दोनों वर्गों के योग के बराबर होता है, जो दोनों भुजाओं पर खीचें जाएँ।

इसमें दीर्घचतुरस्र का अर्थ-आयत (Rectangle), अक्ष्णया रज्जु का अर्थ-तिरछा नापना, एक कोने से तिरछे दूसरे कोने तक नापना (Diagonal), तिर्यङ्मानी (चौड़ाई), पार्श्वमानी (लंबाई)।

यदि एक आयत को बीच में से तिरछा काटा जाय तो उसमें से दो विषम बाहु त्रिभुज निकालेंगे। दोनों का पृथक्-पृथक् जितना क्षेत्रफल निकलेगा, उसका योग ही आयत का क्षेत्रफल होगा। इस प्रकार एक आयत दो त्रिभुजों का जोड़ा समझना चाहिए।

कात्यायन शुल्बसूत्र में इस बात को दो अन्य सूत्रों द्वारा स्पष्ट किया गया है कि यदि लम्बाई और चौड़ाई ज्ञात हो तो कर्ण (Hypotenuse) क्या होगा।

1. यदि चौड़ाई 1 हो और लंबाई 3 हो तो इसका कर्ण 10 का वर्गमूल होगा।

   $1^2 + 3^2 = 10$ के वर्ग का वर्गमूल

   अर्थात् $1 \times 1 = 1$ और $3 \times 3 = 9$, इस प्रकार $1 + 9 = 10$

2. इसी प्रकार यदि किसी आयत की चौड़ाई 2 पद और लंबाई 6 पद होगी तो उसका कर्ण 40 का वर्गमूल होगा। 2 और 6 का वर्ग निकालकर जो जोड़ होगा, वह कर्ण का वर्ग होगा।
   $2^2 + 6^2 = 40$
   अर्थात् $2 \times 2 = 4$, $6 \times 6 = 36$, $4 + 36 = 40$

शुल्बसूत्रों में कर्ण के लिए 'करणी शब्द' का प्रयोग है। शुल्ब और शुल्व शब्द यह दोनों प्रकार से लिखा जाता है।

(क) पदं तिर्यङ्मानी त्रिपदा पार्श्वमानी तस्याक्ष्णया रज्जुदर्शकरणी।
(ख) एवं द्विपदा तिर्यङ्मानी षट्पदा पार्श्वमानी तस्याक्ष्णया रज्जुः चत्वारिंशत् करणी।

कात्यायन शुल्ब0 2.8 - 9

आर्यभट ने इस बात को और अधिक स्पष्ट रूप में लिखा है कि :

यश्चैव भुजावर्गः कोटीवर्गश्च कर्णवर्गः सः। **आर्यभटीय, गणिपाद, 17**

अर्थात् समकोण त्रिभुज में भुजा (Base) के वर्ग और कोटि (Perpendicular) के वर्ग का योग कर्ण (Hypotenuse) के वर्ग के बराबर होता है।

यह नियम **पैथागोरस प्रमेय** के नाम से जाना जाता है। पैथागोरस का प्रमेय है -

"That the square on the hypotenuse of a right-angled triangle is equal to the sum of the square on the other two sides."

अर्थात् समकोण त्रिभुज के कर्ण के ऊपर खींचा गया वर्ग दो समकोण त्रिभुजों के वर्ग के बराबर होता है।

यह पूरा नियम बौधायन, आपस्तम्ब और कात्यायन शुल्बसूत्रों के पूर्वोक्त उद्धरणों से स्पष्ट है। इससे ज्ञात होता है कि पैथागोरस से पहले वैदिक काल में ही यह नियम भारतीयों को ज्ञात हो चुका था।

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SAILESH DAS GUPTA AND VEDIC MATHEMATICS

S. L. Singh

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Sailesh Das Gupta was born on 25 Feb. 1915 in his native place Bangladesh. His father Suresh Chandra Das Gupta was Supdt. of Excise Under Bengal Govt. He prosecuted his early study in Bangladesh and passed the Matriculation Examination with four letters of distinction and secured District Scholarship in 1931. On coming to Calcutta he was admitted in the presidency College, from where he passed I.Sc Examination in 1st Division and B.Sc. with Honours in Mathematics. Presidency college awarded him Kunja Behari Basak Medal for his achievement in B.Sc. Examination. In 1937 he passed M.Sc. Examination in Applied Mathematics of Calcutta University and was placed in the 1st Class. At that time the renowned Scientist Dr. P.C. Mahalanabis started his Indian Statistical Institute at the Presidency College with three versatile mathematicians. Sri Das Gupta joined the Institute as research assistant under Dr. Raj Chandra Bose to work on Industrial Statistics.

After due completion of the course he joined newly formed Sindri Fertiliser as chief statistician. Afterwards he shifted to Britannia Eng. Co, manufacturing firm of Tea and Jute machineries, as chief statistician. During his service there, he contributed the following articles.

1. The Tea Machinery Manufacture in India : Statesman August 1957
2. Production of Jute Machinery in India : Statesman December 1957
3. Growth of Tea Industry in India : Statesman November 1958

In 1958 he joined Martin Burn - a famous industrial firm including The Indian

Iron and Steel Co. as Chief Statistician. He worked with great reputation in dealing with the Tariff Commission Govt. of India for fixation of Steel prices. He collected detailed informations of steel industry of India and abroad and soon became the resourceful person of the world steel Industry. During his service period he contributed sixteen articles on various aspects of steel industry in Economic Times which were greatly appreciated. On his retirement in 1975, he wrote a book under the title' steel prices in India' which remained unpublished. For few years he was associated with some firms at Bombay, Bangalore and Madras for Market Research of Industrial Products.

Since 1980 upto this day Sri Das Gupta seriously engaged himself in the study of Hindu Mathematics in depth. He is alo interested in various other mathematical disciplines. He contributed more than twenty articles on such topics and published in reputed journals. On an assignment, he contributed to Encyclopaedia Asiatica with 'The History of Ancient Mathematics in 'Asia'.

Three books have been published under his authoriship. N.C.E.R.T. - a Govt. of India organisation published π - An unending story in Mathematics. The book has undergone second edition. 'Hindu Ganit 0 Bhaskaracharya, written in Bengali was awarded Rabindra puraskar in 1992 and it is running the second edition. The Third book 'Magic Square' which deals the topic in details including Hindu System, Arabic System and European System of Magic squares.

Presently the voluminous book 'Bhaskaracarya and The Hindu Mathematical Tradition' and illustrated book 'The story of yard and Metre' narrating the history of Measures since the days of Harappa Civilization and Egyptian Civilization to the evolution of Metric System, are with the Publishers.

Sri Das Gupta has also availed the modern communication system i.e. Television. He gave more than ten talks on Hindu Mathematics in Calcutta Doordarshan. Under University Grants Commission Delhi, E.M.R.C. Calcutta produced seven videos with his programmes on various aspects of mathematics and were shown repeatedly since 1994. Sri Das Gupta is now running eighty five years of age, and still active.

Books

1.	Π - an Unending Story in Mathematics	:	National Council of Education and Reasearch, Govt. of India, New Delhi, 1990
2.	Hindu Ganit 0 Bhaskaracarya*	:	Best Books, Calcutta 1992

Rabindra puraskar was awarded to the Author for this book by Government of West Bengal in 1992.

3.	Magic Square	:	Institute of Arts & Crafts, Calcutta, 1996

4. Bhaskaracarya and the Hindu Mathematical Tradition

A detailed book with more than 500 pages on contribution in mathematics by the Hindu Mathematics. The book highlights and proves the number system, arithmetic, algebra and trigonometry first flourished in India and Then went to the Western World Through Arabian Writers : The book is with publisher.

5. Story of Yard & Meter

An illustrated book on measure from Harappa civilization to the evolution of Metric System. The book is with the publisher.

U.G.C. Video Programme by E.M.R.C. Calcutta

1.	π an Unending Story in Mathematics	:	Shown on 20.5.94, 29.12.95, 27.4.96, 28.3.97, 29.7.97, 19.8.97
2.	Transformation Geomatry	:	Shown on 26.4.96, 29.11.96, 10.5.97
3.	Reflection	:	Shown on 7.3.97, 4.10.97, 25.12.97, 8.1.98
4.	Translation	:	Shown on 14.3.97, 11.10.97, 15.1.98
5.	Search for Prime	:	Shown on 19.7.96, 4.10.96, 12.10.96, 5.4.97, 24.11.97
6.	The Story of Yard & Meter	:	Shown on 1.11.97, 16.12.97
7.	Rotation		

Articles

1.	History of Ancient Mathematics of Asia	:	Encyclopaedia Asiatica
2.	Geomatry Through the Ages	:	A.I.M.T. 1998 Vol. 24
3.	Hindu Samkhya Paddhati	:	Aryabhata Vol. 1 1998
4.	Afterthoughts of Wiles' success	:	A.I.M.T. 1997 Vol. 23
5.	Scripts on Search of Prime	:	A.I.M.T. 1995
6.	The Story of Prime Number	:	Ganita Bharati, Delhi 1994
7.	Fermat's Last Theorem	:	A.I.M.T. 1994
8.	Parisamkhayaner ltibritta*	:	Sahityika 1994
9.	Give Me a Right Name (on the names of steel products)	:	Steel Scenario 1993
10.	Mathematician Asutosh	:	Calcutta mathematical Society 13 (1) 1990 Lecture delivered on 125th birth Centenary of Sir Asutosh on 6 September, 1989
11.	Mathematician Bibhuti Bhushan	:	Lecture delivered on birth centenary of Dr. Bibhuti Bhushan Datta on 28 June, 1988. This has been included in the biography of Dr. B.B. Datta
12.	Jyamiti - Mouchak Theke Mahakase*	:	Desh 4 and 11 April 1987
13.	Heath and his blast Furnaces	:	Economic Times 7 June, 1987
14.	Atha Bhaskaracarya Katha*	:	Desh, 28 December, 1985
15.	Map Joker Itihas*	:	Desh, 4 May, 1985
16.	Story of Yard and Metre	:	Indian Journal of Mathematics Teaching 1985
17.	Steel Freight Equalisation	:	Economic Times 30th Oct., 1984
18.	Norms of Wage Fixation (Steel Industry - India)	:	Economic Times 22 Sept., 1984
19.	Steel Prices in 3 Parts	:	Economic Times - Aug. 16, 17 & 18, 1977

20.	**Steel Making: Growth of Blast Furnance and Prospects** :	**Economic Times - Aug. 17, 1973**
21.	**Growth of World Steel Production** :	**Economic Times Oct. 18&19, 1972**
22.	**Mini Steel Plants - Status & Prospectus** :	**Economic Times Nov. 3, 1971**
23.	**Mini Steel Plants in India Prospects & Possibilities** :	**Iron & Steel Review Aug., 1973**
24.	**Impact of Excise Duty on Iron & Steel** :	**Economic Times May 2, 1971**
25.	**Growth Pattern of Wrold Steel** :	**Martin Burn House Magazine - 1971**
26.	**The Kaldo process in Modern Steel Making** :	**Economic Times Aug. 5, 1965**
27.	**Iron Ore Production in India** :	**Economic Times Jan. 23, 1964**
28.	**Production & Consumption of Iron Ore in India** :	**Economic Times Jan 23, 1964**
29.	**Coal will remain vital fuel as ever** :	**Economic Times Jan. 19, 1963**
30.	**Measurement of Blast Furnace Efficiency** :	**Economic Times April 3, 1962**
31.	**Blast Furnace in India - A Review** :	**Economic Times February 13, 1962**
32.	**Productivity Measurment of Blast Furnace** :	**Economic times May 5, 1961**

33.	Growth of Tea Industry in India	:	Statesman Nov. 1958
34.	Production of Jute Machinery in India.	:	Statesman December 1957
35.	Tea Machinery Manufacture in India	:	Statesman Aug. 1957

In addition to the list.

36.	Aryabhata The Father of Hindu Mathematics	:	International conference on History of Mathematics
37.	Ganitacarya Asutosh*	:	Adhunik Ganit Anwesa
38.	Salaka - purusa Asutosh	:	Golden Jubilee of presentation of Asutosh collection to National Library celebrated by the National Library
39.	Many Faces of Lear's Carroll	:	A.I.M.T. Vol 24 1998

* In Bengali

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Abstracts of Papers/Talks

INTERNATIONAL CONFERENCE on HISTORY OF MATHEMATICS

December 16-19, 1999

Department of Mathematics and Statistics

GURUKULA KANGARI VISHWAVIDYALAYA

HARDWAR 249404

1
THOMAS S. KUHN & THE HISTORY AND PHILOSOPHY OF MATHEMATICS

Sasaki Chikara

University of Tokyo, Japan

It is no doubt that Thomas S. Kuhn tranformed drastically our view of science since the publication of *The Structure of Scienfific Revolutions* in 1962. Kuhn himself called his new vision of science "the historical philosophy of science". As most of historians of mathematics may know, the problem of whether or not his view of science is applicable to the case of mathematics has been discussed to a considerable extent. The beginning of recent arguments was marked by Michael J. Crowe's essay "Ten 'Laws' Concerning Patterns of Change in the History of Mathematics," published in *Historia Mathematica*, Vol.2 (1975). The tenth 'law' by Crowe has stated categorically that "Revolutions never occur in mathematics". Attemts to refute this 'law' has followed by, for example Joseph W. Dauben's "Conceptual Revolutions and the History of Mathematics: Two Studies in the Growth of Knowledge", published in 1984. By taking examples from Eudoxus's theory of proportion applicable to both commensurable and incommensurable quantities and George Cantor's transfinite set theory, Dauben has proposed a new thesis contarary to Crow's "Revolutions have occurred in mathematics". Debates on revolutions in the history of mathematics may be seen in a collection of papers edited by Donald Gillies, *Revolutions in Mathematics* (Oxford: Clarendon Press, 1992). Recently, another collection of essyas on the same topic was published by Elena Ausejo and Mariano Hormigon under the title *Paradigms and Mathematics* (Madrid: Siglo XXI de Espana Editores, 1996).

As a student of Kuhn at Princeton University, I attended his course on the philosophy of science and sumitted to him a paper on revolution in mathematics in 1977. Before that time, Kuhn had not thought that his theory of science is applicable

to mathematics and this view certainly influenced Crow's essay of 1975. In his comment on my paper, Kuhn had changed his previous view and wrote clearly that "There must have been revolutions in mathematics". This is a remarkable statement.

Thus, my present lecture is arguing favorably for Dauben and trying to present "the historical philosophy of mathematics", as it were, an extension of Kuhn's "historical philosophy of science". For this purpose, first of all, our understanding of mathematics should be rivised. The so-called 'Platonic' or formalist understanding of mathematics must be rejected and a new conception of mathematics should be introduced, for example, mathematics as a "quasi-empirical" science, to use Imre Lakatos's terminology. Moreover, historians of mathematics so far seem to have discussed the problem, having taken examples from the history of mathematics of modern Europe, at most from that of ancient Greece and modern Europe. In addition to the history of mathematics in Europe, however, we have to examine further how mathematical culture in East Asian countries which enjoyed a rather high standard was transformed from traditional to the one based on modern Western mathematics. In this sense, our scope of discussion should be extended from the history of Western mathematics to the history of mathematics in the world.

Consequently, we cannot deny any longer that there have existed revolutions or drastic transformations` in mathematics. I distinguish three types of change in the history of mathematics:(1) Merely evolutional changes; (2) Revolutionary changes in basic conceptions and mediae which Dauben has analyzed; (3) Drastic transformations associated with shifts of fundamental social and cultural structures, as exemplified in the history of mathematics in East Asian countries.

2
THE HISTORY OF THE 'SONA' GROMETRICAL TRADITION IN CENTRAL AND SOUTHERN AFRICA

Paulus Gerdes

Universidate Pedagogica, Rua Macroni 109 R/C, Maputo, Mozambique

The 'sona' sand drawing tradition developed in northeastern Angola and neighbouring areas of Zambia and Congo/Zaire. An analysis and reconstrution of mathematical elements in this tradition will be presented. Particular attention will be given to the cultural values of symmetry and monolinearity of the designs, to classes of 'sona' and the geometrical algorithms for their construction, and to composition and chain rules for monolinear 'sona', and to the possibility of reconstructing lost classes. 'Parallel' developments in other parts of Africa and in other periods and continents will be indicated, e.g. Ancient Egypt, Ancient Mesopotamia, 'kolam' designs in South-India.

Books by the author dealing entirely or partly with 'sona' geometry will be on display:

* "Sona Geomety: Reflections on the sand drawing tradition of peoples of Africa south of the Equator", Universidade Pedagogica, Maputo, 1994, Vol. 1, 200 pp.;
* "Une tradition geometrique en Afrique.- Les dessins sur le sable" (3 volumes), L'Harmattan, Paris, 1997, 127 pp.;
* "Recreations geometriques d'Afrique- Lusona- Geometrical recreations of Africa", L'Harmattan, Paris, 1997, 127 pp.;
* "Ethnomathematik dargestellt am Beispielder Sona Geometrie", Spektrum Verlag, Berlin/Heldelberg/Oxford, 1997, 436 pp.;
* "Geometry from Africa: Mathematical and Educational Explorations", The Mathematical Association of America, Washington DC 1999, 210 pp.

3

MATHEMATICS IN THE MAGHRIB XIth-XVIthc- AN OVERVIEW

Abdelkhalek Cheddadi

Mohammedia School of Engineering, Mohamed V University, Rabat, MOROCCO

Compared to Mathematics in the eastern part of the Muslim World, the history of Mathematics in the Maghrib is quite little known. Although scarce biographical information exist concerning mathematicians from ages as early as the IXth or even the VIIIth c.- especially from Aghabid Quayrawân - it seems that the expansion of Mathematics in the Magrib is strongly indebted to the andalusian scholars, from the Xth c. onward

A turning-point came in 1085, with the fall of Toledo into the hands of Alfonso VI of Castile and the subsequent political unification of Morocco and al-Andalus carried out by the Almoravid Yûsuf ibn Tâshfîn. Closely related to the development of rational sciences in Islamic Spain, Mathematics flourished in Marocco under the Almohads (XIIth-XIIIth) who ruled the entire Islamic West. During this era, the highly sophisticated wealth of Andalusian scientific knowledge spread to Marrakesh, Fez, Tlemcen...

A specific mathematical tradition inaguranted by al-Hassâr[1] and Ibn al-Yâsamin was continued and firmly established by Ibn Qunfudh and later the Grenadin al-Qalasâdî and Ibn Ghâzî of Meknes who rank among the most famous mathematicians in the Maghrib between IXth and XVIth c.

Recently, manuscripts from different areas- including some of recent discovery- have ben studied, and now allow a relatively accurate knowledge of the develoment of Mathematics in the Islamic West[2,3].

1. A. Cheddadi : A Problem of Arithmetic and its Solution by the Moroccan Mathematician al-Hassar (XIIthc.), International Congress of Mathematicians, Berlin, August 18-27, 1998.
2. A. Djebbar: Enseignement et Recherche mathematiques dons le Maghreb des XIIIth-XIVth siecles, Publications Mathematiques d'Orsay, Paris, 1981.
3. Reccent bibliography in J.L. Berggren: Mathematics and Her Sisters in Medieval Islam, Historia Mathematica, vol. 24, 1997, pp.407-440.

4

THE GREGORIAN CALENDER : A SCIENTIFIC MASTERPIECE OF THE LATE RENAISSANCE

Heiner Lichtenberg

Diplom-Mathematiker, Otto-Hahn-Str. 28, D-53117 Bonn-Buschdorf

The Gregorian Calendar system, now the de facto worldwide standard, is often mistakenly thought to be a purely solar calendar. However, since its inception in 1952, it has in fact been a lunisolar timekeeping system. The Gregorian calendar relates dates to the sun and the moon simultaneously. The reason for this is the Christian feast of Easter, which has been defined by the Council of Nicaea in 325 A.D. as the first Sunday after the first full Moon on or after the vernal equinox (of the northem hemisphere). The fathers of the Gregorian calendar are Aloysius Lilius (died 1576) and Christophorus Clavius (born 1537, died 1612).

A calendar dates every day. Therefore the Gregorian calendar dates every day not only by the common solar date, known to all of us, but aslo by a lunar date, known to almost no one. This lunar date gives the phases of the moon. The first month of the lunar year is the spring month named Nisanu by its babylonian name. The fourteenth day of every moon month denotes the full Moon. With these explanations the definition of Easter Sunday becomes : Easter Sunday is the first Sunday after the fourteenth day of Nisanu.

The structure of the lunar side of the Gregorian calander will be shown in detail for the centuries 20 to 22. There are ten types of lunar years, especially four types of lunar common year with 12 moon months and six types of lunar leap years with 13 moon months. For keeping the lunar years in step with the tropical years the so-called Callippian cycle (Callippos from Kyzikos, about 330

B.C.) has used: 76 tropical years = 940 synodical months = 27,759 days.

The Gregorian calendar is an adaptable time-reckoning systems. This means that if the sun and the moon change their mean velocities the Gregorian calendar is able to follow within a certain frame. The determination of this frame will be given form the so called calendar equations :

$$a\_trop = 1461 / 4 - \sigma / (100.P_1)$$

$$m\_syn = a\_trop / (235 / 19 + \varepsilon / (3000.P_2)$$

In these equations *a_trop* means the length in days of the mean tropical year and *m_syn* the length in days of the mean synodical month of the Gregorian calendar. σ is the number of leap years reverting to common years at secular boundaries of the period P_1, measured in centuries, and ε is the number of (net) adjustements of the epact at the secular boundaries of the period P_2, mesured in centuries.

As a conculsion of the first calendar equation I will demostrate how the secular leap rule in use (leap year, if the number of the secular year is divisible by 400, otherwise common year) can be derived from the natural value for *a_trop* by a continued fraction.

Finally I discuss steps towards a full mathematical theory, in modern terms, of the Gregorian calendar, this scientific masterpiece of the late Renaissance.

H. Lichtenberg, The Gregorian Calendar: An Adaptable Cyclic Lunisolar Time-reckoning System for the Millenia, Human Welfare Studies of Hokkaido Women's Univ., vol. 2 (1999), pp. 137-148.

5
THE DEVELOPMENT OF THE WORLD RESEARCH COMMUNITY : THE GENEALOGY PROJECT

Harry B. Coonce

Minnesota State University, Momkato, U.S.A.

6
PLACE OF RAMANUJAN IN THE HISTORY OF MATHEMATICS

Jamuna Prasad Ambasht

Dep. of Mathematics and Computer Sceince

34, Westpine Court Columbia, S.C. 29212, U.S.A.

7
THE ILLUSION THAT SCIENCE ADVANCES BY REVOLUTION FOLLOWED BY PARADIGM CHANGES

P. R. Masani*

Pittusburg, U.S.A.

***[His recent sad demise is deeply condoled.]**

This paper systematically demolishes Kuhyn/s thesis by reference to 3 so-called revolutions, the Copernican, the Relativistic, and the Quantum Mechanical. The second is the easiest to deal with, since Einstein categorically denies having made any revolution. The first is very interesting, but involves a digression on the impact of Aristotle on Greek science. The last one is the most difficult and the one in which I am the most interested. Each "revolution" involves high-powered mathematics (even the first, when we think of Kepler), and falls into the rubric "History of Mathematics".

8

HISTORY OF MATHEMATICS IN THE CONTEXT OF NEPAL

S. R. Pant

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9

ĀRYABHAṬA THE FATHER OF HINDU MATHEMATICS

Sailesh Dasgupta

70, Purna Mitra Place, Calcutta

In 1976 India celebrated the 1500th birth anniversary of Āryabhaṭa in a grandoise fashion. Āryabhaṭa composed his masterpiece Āryabhaṭia- a book on mathematics and astronomy in 499. Anong the Hindu extant mathematical texts. Aryabhata is the oldest. We have assembled here to commemorate the 1500th anniversary of the composition of Aryabhatia.

The great mathematician and astronomer Āryabhaṭa (476-550) is to-day know to all. Though no biography of the great sovant exists, yet we know that he was born in 476 A. Dand wrote his famous book Āryabhaṭia when he attained the age of 23. He studied at Kusumapura- modern Patna. This scanty information is gathered from his book.

However from his commentator, we know he was a Kulapati or Head of an institution. Bhaskara I his principal commentator callls him sarva-siddhanta-guru. His disciples formed a new school of astornomy called 'Āryabhaṭa School'

Āryabhaṭīya is a small compendium of mathematics and astronomy and meant for his students. The book contains only 121 slokas and written in

poetic form. The book is divided into four parbas gitikapado (23 slokes) ganitapada (33 slokas) kalakriya pada (25 padas) and gdopada (50 slokas). His main contributions are :

1. Throughout his book he used Hindu numerals with sunya.
2. Alphabetical systen of numeration.
3. Formation of sine-table which is recognised as first table in mathematics.
4. Methods for extracting square and cube roots.
5. Theory of Earth's daily rotation.
6. Determination of positions and motions of heavenly bodies.
7. First determination of accurate value of π by trigonometrical method.
8. Devising method for solution of Indeterminate equation of first degree.

10
ELEMENTS OF MATHEMATICS IN VEDIC LETERATURE

***Padmshri* Kapil Dev Dvivedi**

Director, Vishwa-Bharti Reasearch Institute, Gyanpur 221304

(See Pages 9-35 for the full article)

11
A TALK ON ANDRE WEIL'S INDIA IN EARLY THIRTIES

B. S. Yadav

TU-67, Vishakha Enclave, Delhi

12
DEVELOPMENT OF ASTRO-MATHEMATICAL SCIENCES IN ANCIENT BIHAR

Parmeshwar Jha

Saraswati Sadan, Vidyapuri, Supaul (Bihar)

Bihar played an important role for the development of India thought and culture in different aspects. Several attempts have been directed in the past to take stock of philosophical and allied literatur produced in the land, but very little efferts have been made to bring to light its mathematical and astronomical literature. Consequently contributions of ancient Bihar in the field could not yet be preperly asssessed. The present paper is an attempt in this direction. The paper explores some of the mathematical results and astronomical principles contained in religious and canonical works produced in Bihar right from the 10th century B.C. (cirea) to the period of Puranas. It has been shown with sufficient ground that the works of Yajnavalkyn (10th Cen. B.C. - 6th Century B.C.), Buddhist literature, Jainn Canonical works, Kautilya's Arthshastra and such other works of ancient Bihar consist of several mathematical principles and astronomical Constants. The paper deals with different aspects of the Ganita, Jataka and Samhita Skandhas of Hindu astrenomy as found in Puranns especially composed in ancient Bihar. It has also been discused that a school of mathematics and astronomy was in existence at Kusumapura (Patna, the modern capital of Bihar) which was nourished by several scholors including Aryabhata (b. 476 A.D.) who was one of the greatest exponents of this school and laid the foundation of different branches of mathematics. It has also been shown that the school contained unabated for several centuries and produced a number of great scholors in mathematics and astronomy.

13
INVENTION OF ZERO AND DECIMAL PLACE VALUE SYSTEM-A SUPREME CREATION OF HINDU MATHEMATICS

V. M. Shah

Dep. of mathematics, Faculty of Science, M.S. University of Baroda

14

LIMIT CONCEPT IN ANCIENT INDIAN MATHEMATICS

S. L. Singh

Department of Mathematics & Statistics, GKV, Hardwar

The purpose of this talk is to give a development of the limit concept in ancient Indian Mathematics. It is concluded that Bhaskara II is the first to give a correct presentation of the limit in his Līlāvatī and Bījagaṇita.

15

ON NONLINEAR FUNCTIONAL EQUATIONS INVOLDING DISCONTINUITIES

B. C. Dhage

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In this lecture a survey of the historical development of the results concerning the solution of the nonlinear functional equations *fx=x* will be taken in a certain topological space. Unlike most of the results in topological spaces, a set of sufficient condition will be given for the existence of the solution of four simultaneous nonlinear functional equations under weaker condition of the continuity. Finally some open problems in this direction will be discussed.

16

YEAR AND PLACE OF BIRTH OF ARYABHATTA REFIXED

K. K. Velukutty

College of Arts and Science, Sri Ramakrishna Mission Vidyalaya, Coimbatore

Āryabhatta, author of Āryabhaṭīyam, Baskara I, author of Leghubaskariyam and Mahabaskariyam and Haridtta, author of Grahacharanibhandana and

Mahamarganibhandana. The teacher and two of his disciples in Mathematics, are identified with Vararuchi, Pakkanar and Naranathubranthan of Parachipettapanthirukulam and with three among Navaratnas of Kulasekara I, Keralavikramadhitya of Chera dynasty. The place of birth of Aryabhama is Padaliputhra of South India, a township in Asmaka which was once the southernmost province of Chandragupta. Āryabhatta is "Asmakajanapadhajaatha" (Neelakandasomayaji). Now Asmaka is identified in Arcot in Tamil Nadu. His year of birth is 581 A.D. instead of 476 A.D. Āryabhatta is no one but Kumarilabhatta I of Sanskrit literature and Mochikeeranar of Tamil literature. On the way of fixing year and place of birth of Aryabhatta, the hypothesis of the principle of "three phases and three addresses' of scholars belonging to Cheranadu is introduced. The scholars of then Cheranadu owned 'three phases and three addresses': one in Sanskrit, one Tamil and one in Malayalam. It is not exactly three but atleast three: Kumarilabhatta I- Mochikeeranar - Vararuchi - Āryabhatta - Sukumarakavi, Jajjadasishya - Pilantholmoosathu - Pakkanar - Baskara I, Haridatta - Ponmudiyar - Naranathubramhah, Mandanamisra - Vanparanan - Mezhathol Agnihotri, Dharmakeerthi - Thiruvalluvar - Vallon, Salikanathan - Arasikeezharu - Thiruvankarathupanan, Bhattanarayana - Umbekan - Vayillakunnilappan, Bharthruhari - Yelidana - Bhatti, Agastya - Kurumuni - Avilokideshwara, Athulan - Kapilar - Tholan- Villumangalathuswamiyar and so on. Also Thiruvalluvar, father of Tamil literature is the son of Bharthruhari, Kumarilabhatta I, Kulasekara I, Agnihotri and Bharthruhari are the sons of a Brahmin born to four wives from Brahmin, Kshatriya, Vysya and Sudra respectively. This chapter of history of South India is intimately linked to Parachipettapanthirukulam of Kerala and Navaratnas of Keralavikramadhitya and the men Kalinga Empire. The conclusions continue.

Ref: K.K. Velukutty, Heritages to and from Aryabhatta, Sahithi, 1997.

17
PRE-HISTORIC ORIGIN OF THE COVER-UP RULE FOR THE PARTIAL FRACTION

Baikunth Prasad Ambasht

Engineering College, Bokaro Steel City

In this paper we study the Vedic origin of the so-called cover-up rule for the partial fraction attributed to Heaviside by European scholars. Recall that in Berard and Child algebra book: "Express

$$\frac{x^3}{(x-a)\,(x-b)\,(x-c)}$$

as the sum of a constant and three proper fractions and hence or otherwise prove that the identity involving a,b,c,d holds in the symmetrical form

$$\frac{a^3}{(a-b)\,(a-c)\,(a-d)} + \frac{b^3}{\ldots\ldots} + \frac{c^3}{\ldots\ldots} + \frac{d^3}{\ldots\ldots} = 1."$$

The elegant cover-up is known to Vedic Scholars as "VILOPAN" which translates from Saṃskrit into "disappearance".

18
SOME ASPECTS OF THE CULTURAL HISTORY OF MATHEMATICS

K. S. Chaudhuri

Department of Mathematics, Jadavpur University, Calcutta

The purpose of this talk is to explore the linkage between mathematics and humanistic culture over the centuries of human civilisation. It will be discussed how mathematics influenced the society and culture in the Indian sub-continent upto the 12th century A.D. and in the western Europe in the 17th century A.D. The reasons for gradual erosion of this link between mathematics and culture from 18th century onwards will be discussed.

19
GEOMETRICAL TREATMENT OF PROGRESSIVE SERIES BY SANKARA

V. Madhukar Mallayya

TC 25/1975 , Deshabhimani Road, Trivandrum, Kerala

Geometric representation of series is a special feature of Indian Mathematics which is probably an outcome of religious needs for construction of vedic altars. Āryabhaṭa's terminology like citi for sum of series shown that representation of series using piles was known to him. Both Ñilakanṭa in his Āryabhaṭīya bhāṣya and Śaṅkara in his Kriyākramakarī have used the sort of representation AP using piles. Śaṅkara has given a fine geometrical representation of GP and has also established a recurrence relation on the background of geometry. While the relation of series with geometry is latent in Āryabhaṭa's terminology, the full bloom of such geometrico-algebraic imagination can be seen in Śaṅkara's Kriyākramakarī.

20
THE CONCEPT OF GENERALIZED FUNCTIONS IN VEDANT PHILOSOPHY

G. S. Pandey

Dep. of Mathematics and Astronomy, Lucknow University, Lucknow

It is beyond dispute that the concepts of zero and infinity were the original contributions of ancient Indian scholars. The mathematical significance of zero was fully understood even before 400 B.C. The Vedant Philosophy is mainly concerned with the relationship between God (infinity) and Soul and has developed the theories of the immortality of soul, rebirth and liberation from the bondage of this physical world.

As pointed out by James Jeans, the philosophy of any period is largely interwoven with the science of the period. In the present talk it is proposed to study the mathematical aspects of Vedant Philosophy and its relationship with the theroy of generalized functions developed by Laurent Schwartz and I.M. Gelfand and G.E. Shilov. Also, using the notion of generalized functions, we provide viable mathematical proofs of some outstanding theories of Vedant Philosophy.

21
A SEARCH FOR ABSOLUTE ZERO OF HISTORY OF MATHEMATICS

Aloke Nath Sensharma

Visva-Bharti Basantika-Begampara, Santiniketan

22
A DEVELOPMENT OF JUNCK'S CONTRACTION

R. Arora* B. Ram and S. S. P. Singh

Dep. of Mathematics, Tehri Campus of H.N.B. Garhwal University, Tehri

**Himalayan Institute of Medical Sciences, Dehradun*

Jungck's fixed point theorem for a contractive type pair of maps on a metric space (1976) was first generalized by S. L. Singh (1977). Thereafter a huge theory developed for jungck type contractive maps. The purpose of this paper is to highlight same.

23
HISTORY OF BOUNDARY LAYER

T. C. Panda and S. K. Mishra

Dep. of Mathematics, Berhampur University, Berhampur, Orissa

This reiew deals with history of the development of boundary layers starting from Prandtl's boudary layer theory [1904] until the latest development in boundary layer in various fields of Physical processes like Cyclone, Weather Prediction, Environmental Pollution, aerospace dynamics etc. The subject matter is focused on Genesis, earlier developments, Prandtl's paper, prototype concept, Steady and Unsteady two dimensional Velocity and thermal boundary layers, Three-dimensional boundary layers, Compressible boundary layers, Instability and transition to turbulence, Higher order boundary layer, Boundary layer separation, Boundary layer control, Two phase boundary layers, Turbulent boundary layers, Prandtl's mixing length theory, *K*-theory, Ist order, $1\frac{1}{2}$ closure, higher order closure, K-ε model, l-ε model, Atmospheric boundary layer, Laminar and Turbulent Ekman layer, Surface layer, dynamic sublayer, viscous sublayer; similarity theory for the surface layer and atmospheric boundary layer, semi-empirical theory of the atmospheric boundary layer, Synoptic scale, Meso-scale and micro scale modelling in the atmosphere.

24
CONTRIBUTIONS OF NĀRAYĀNA PANDIT TO THE SOLUTIONS OF THE EQUATIONS OF THE TYPE $NX^2 \pm 1 = Y^2$ {i.e. SQUARE NATURE (VARGA-PRAKṚTI)}

Rama Shankar Lal & Ramasheesh Prasad*

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**G.M. High School, Barharia, Siwan (Bihar)*

The term Varga-prakṛti or Kṛti-Prakṛti has been issued by the Hindus to designate the equations of the type $NX^2 \pm C = Y^2$. The most fundamental equation of this type is $NX^2 \pm C = Y^2$, where N is a non-square integer.

Brahmagupta (628 A.D.) has established two important lemmas before proceeding to the general solution of square nature. Śripati (1039 A.D.), Ācārya Jayadeva (1050 A.D.), Bhaskara II (1150 A.D.), Nārāyana Pandit (1350 A.D.), Jñānarāja (1503 A.D.) and Kamalākāra (1658 A.D.) have given rules for the solution of the above equation. Here effort has been made to bring into light only the work of Nārāyana Pandit to the Topic.

25

ORIGIN AND DEVELOPMENT OF INDIAN MATHEMATICS

Bhartendu Dvivedi

Govt. P.G. College, Hamirpur (U.P.)

26

ON NONLINEAR DISCONTINOUS SECOND BOUNDARY VALUE PROB LEMS

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Gurukula Colony, Ahmedpur, Latur (Maharashtra)

**Marathwara Univ., Nanded*

Let R donote the real line, $R+$, the set of all non-negative real numbers. Let $a, b \in R$, $a<b$ and let $J=[a,b] \subset R$ be a closed and bounded interval in R. Now consider the non-linear two point second boundary value problems (in short *BVP*) of ordinary second order differential equations,

$$\left.\begin{aligned} -x'' &= f(t,x) \ a.a.t \in J \\ x(a) &= 0 = x'(b) \end{aligned}\right\} (*)$$

where $f : JxR \rightarrow R+$ is a function.

In this paper the historical survey of the known work on the second *BVP* (*) will be taken with continuous and discontinuous nonlinearity f. Further the existence of the externmal solutions of *BVP* (*) will be obtained via a known fixed point technique due to Tarski. Finally two differential inequalities will be obtained which will be applied to the *BVP* (*) for proving the boundedness and the uniqueness of the solution to the *BVP*(*).

27

VEEZANKA AND SOME CONCEPTS IN MODERN ALGEBRA

Sandeep Kumar Bhakat

Siksha-Satra, Visva-Bharati, Sriniketan (W.B.)

Ancient Indian Mathematics was the pioneer in respect of the culture and development of mathematical research in the early century. A lot of important and significant mathematical concepts and formulae had been generated in the Vedic age in India. To study the "natural number system and arithmetical operations" in the early century, there were a lot of "sutras" stated in 'Atharva veda'. To divide the whole natural number system into disjoint classes was a very important task from astrological as well as mathematical point of views. The concept which played key role to partition the natural number system was "veezanka". For any given natural number n, take the sum of the digits of n, if the sum is not a single digit number, then add all the digits and so on till the sum is a single digit. This resultant digit is called the "Veezanka" of the given natural number *n*. For a single digit number, the vezanka of that number is the number itself. According to the definition, the veezankas are *1,2,3,4,5,6,7,8,9.* The whole natural number system is partitioned into nine disjoint classes, called veezanka classes. The set of all veezankas forms a "group" with respect to the operation "veez-sum" (suitably defined). For any two natural numbers *a,b* and *a>b, a ≡ (mod 9)* if and only if veezanka of *a* is equal to veezanka of *b*.

28

CONCEPTS OF ZERO IN JAINA TEXTS

N. L. Jain

12/644, Bajrangnagar, Rewa (M.P.)

The purpose of this paper is to present an account of zero in jaina Cannon and mathematics in ancient India.

29
INDIAN PERCEPTION OF QUADRATIC EQUATIONS

Vinod Mishra and S. L. Singh*

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**Dep. of Mathematics and Statistics, Gurukula Kangri University, Hardwar*

May it be the case of Arabs, Babylonians, Chinese, Europeans, Greeks or others, all except Indians were too much extent unaware of the existence of negative numbers, irrationnal numbers, and more so number and nature of roots of a quadratic equation before17th century A.D. The Indian knowledge in this regard was the peak during the time of Bhaskara II (12th century A.D.) not only in ordinary quadratic equations but also in complex as well as simultaneous quadratic equations. This paper attempts to ascribe the contributions of Indians as regards quadratic equations from the Śulbasūtras to 14th century A.D. of Nārāyaṇa Paṇḍita.

30
THE EMPTINESS-ANCIENT AND MODERN VIEWS

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**Department of Physics, GKV Hardwar*

In Vedic "*Bangmaya*" a great contemplation has been made regarding vacuum or void. It has been asserted that "*Sarwam Khalu Idam Brahmm*" (all this is Brahm i.e. totality, wholeness). In Buddhistic Philosophy also Nagarjun has deeply pondered over vacuum and this philosophy is known as "*Sunyawad*". In modern Physics the subject of vacuum has been dealt with

great detail specially by Dirac. He considered the vacuum as filled with still uncreated electrons. According to him vacuum is not nothingness. In this paper a critical study of the ancient and the modern views has been presented.

31
COMBINATORIAL PROBLEMS IN HINDU MATHEMATICS

V. Mishra and A. S. Grewal*

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This paper briefly discusses 'combination' and 'permutations' concepts widely used in ancient and medieval Indian mathematics. Emergence of theory outside India is also dealt with.

32
MATKOWSKI CONTRACTION PRINCIPLE

U. C. Gairola

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In 1973 Janusz Matkowski extended the concept of Banach contration to a system of equations on a finite product of metric spaces and obtained a fixed point theorem for such a system of operators. Later on this concept is generalized and improved by several mathematicians and named as Matkowski contraction principle (*M.C.P.*). In this paper we will discuss the development of *MCP*.

33
ALGORITHMS FOR THE PERFECTNESS OF NUMBERS

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**Vidya Mandir, Sect. V, BHEL, Hardwar*

In this paper an attempt is made to check the perfectness of powers of a given number. The method indeed may be applied to check the perfectness of n^{th} power of any positive integer.

34
ANCIENT SCIENCE TO MODERN SMART MATERIALS

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To understand the nature of matter Indian "*Vaisesika System*" cosidered the existence of atoms, the smallest particle as with dimension of less than mathematical points. These points possesed potential quality of four elements- earth, water, fire and air. In this development the quest of mankind to find better materials has crossed several distinct stages, starting with stone age, bronze age and latter iron age. In the present paper attempts have been made to show the relevance of ancient views of materials in the modern Physics to cover definition, evolution and characterization of some of the inherent properties of the materials for the development of many new devices and technologies.

35
ASTRONOMICAL CONSTANTS

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The intent of this paper is to explain the Astronomical Constants given by *Bhaskaracarya* (1150 A.D.). The planetary position can be computed for any day with the help of Astornomical Constants.

36
COINAGE IN GAṆITATILAKA

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In this paper we discuss some salient features of coinage in Vedic Mathematics specially in *Gaṇitatilaka*.

37
GEOMETRY IN RITUALS

Vinod Mishra and S. L. Singh*

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Śulbasūtras, the manuals for constructing sacrificial altars of various geometrical designs are the rich sources of mathematical knowledge of Indo-Aryan savants of first millennium B.C. at least. This paper highlights the major contents of the Śulbasūtras.

38
A DEVELOPMENT OF STATISTICS IN ANCIENT INDIA

V. Kumar

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First known use of statistics was in Egypt about 3050 B.C. in census taking. After this in 594 B.C. in Greece and ancient India it was in Mouryan period. Till 16th century the use of statistics was limited to census taking. In sixteenth century Tycho Brabe used this subject for studying movement of astronomical objects. In seventeenth century much work was done in vital statistics by Jhon Graunt, Edmund Hally and Casper Newman. In its present concept subject came into existences in eighteenth century. For this L. A. J. Quetlet is credited. After this subject developed in multy fields. In twentyth century a new branch under the name Econometic came into existence. In this way the subject developed to the present form.

39
A DEVELOPMENT OF SPECIAL FUNCTIONS AND RELATED INTEGRAL TRANSFORMS IN INDIAN SUBCONTINENT

J. M. C. Joshi

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40
REMARKS ON THE ROLE OF SZASZ OPERATORS AND ITS HISTORY

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The purpose of the paper is to discuss some historical aspect of Szasz operators in Approximation theory.

41
SOME CONTRIBUTIONS TO MATHEMATICS IN ANCIENT INDIA

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The purpose of this paper is to discuss the work of Indian mathematicians of antiquity and the status of ancient Indian mathematics.

42
SAMARĀṆGANA SŪTRADHĀR AND MATHEMATICS

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Samarāngaṇa Sūtradhāra of Bhoja is primarily dedicated to architecture. King Bhoja ruled Dhārā Nagarī, a part of central India from 1000 A.D. to 1055 A.D.

The purpose of this paper is to present some Mathematical referances from Samarāngaṇa Sūtradhār.

43
ACARYA SRIDHARA AND MAHAVIRA-A COMPRATIVE STUDY

Anupam Jain

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Acarya Sridhara (750 A.D.) and Acarya Mahavira were two great Mathematicians of India. The credit of composing Patiganita (Navamsati) and Patiganitasara is given to Acarya Sridhara. One more book Bijaganita is also composed by Sridhara but it is not available at present. This list is not complete.

Acharya Mahavira (Mahaviracarya-850 A.D.) was the author of famous Indian mathematical text Ganitasara Samgraha. Four more books Jyotisa Patal Sattinshika, Ksetraganita, Chatisa Purva Uttara Pratisaha are said to be of Mahavira.

The present paper discusses the period and works of both the Acarya in detail. In the 6-7th decade of this Century Sridhara is fixed about 950 A.D. by some historians due to it he became posterior to Mahavira and the credits of inventions of those formulas & Concepts which is found in the works of Sridhara &Mahavira both is given to Mahavira only. Now it has been finaly established that Acarya Sridhara is prior to Mahavira and in light of it a comparative study of the works of both is essential. I have tried to put light on it in the present paper.

A detailed list of the works done on Sridhara and Mahavira is also given to facilitate the further studies.

44
DID VEDIC SAVANTS KNOW IRRATIONAL NUMBERS ?

V. Mishra and S. L. Singh*

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**Dep. of Mathematics and Statistics, Gurukula Kangri University, Hardwar*

This paper presents a brief account of square-root and cube-root methods in Indian perspective. A method for computing $\sqrt{N}$ ($N > 0$, a non square integer), in the form of a Proposition based on iteration is given. Finally, we conclude that the evalutaion of square-rooting in *Śulba* and post-*Śulba* period was most probably based on this Proposition. At the end, a program to calculate square-root of *N* using above Propostion in *C-LANGUAGE* is chalked out.

45
A DEVELOPMENT OF NUMBER SYSTEM IN INDIA

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In the present paper development of numerals in India has been studied. An attempt has been made to investigate those high numbers which were frequently used in ancient India.

46
A DEVELOPMENT OF MATHEMATICS RESEARCH IN HINDI LANGUAGE

D. D. Sharma

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The supervision of Doctoral Thesis (Mathematics) seems to have been initiated by Professor S. L. Singh (Hardwar). The purpose of this note is to present an account of Doctoral Theses (Mathematics) and on going doctoral level work through Hindi Medium.

47
APPLICATION OF MATHEMATICS IN GROUP TECHNOLOGY

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48
A DEVELOPMENT OF C-CONTINUITY IN TOPOLOGICAL SPACES

Suresh Pal

Department of Mathematics & Statistics, GKV, Hardwar

49
A NOTE ON THE DEVELOPMENT OF HYBRID CONTRACTIONS

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**Dep. of Mathematics, Kumaun University, Nainital*
***Research Scholar, GKV, Hardwar*

Nadler's multivalued contraction is of vital importance in nonlinear analysis. The hybrid contraction, first introduced by S.L. Singh and Chitra Kulshreshtha, now attracts a good attention in multivalued analysis and applications. The purpose of this paper is to present a brief development of hybrid contractions.

50
A STUDY OF HISTORICAL DEVELOPMENT OF FIXED POINT THEOREMS FOR FUZZY MAPPINGS

R. C. Dimri

Dep.of Mathematics, H.N.B. Garhwal University, Srinagar (Garhwal)

The notion of a fuzzy set was introduced by L. Zadeh in 1965. Since then an intensive study has been done in the field of fuzzy topology, fuzzy metric, metrizability and fuzzy uniformity. In recent years the fixed point theorems for fuzzy mappings have been studied by many authors. In this note we present a historical development of fuzzy mappings, fuzzy metric spaces and related fixed point theorems.

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A COMPARISON OF TWO CONTRACTION PRINCIPLES

B. C. Dhage and V. V. Dhobale*

Gurukula Colony Ahmedpur, Latur (Maharashtra)

**Mahatma Gandhi Mahavidyala, Ahmedpur, (Maharashtra)*

In this paper a variant of *D*-contraction mapping principle is obtained under the weaker condition of the boundedness of the *D*-metric space.

A comparison of the well-known Banach Contraction mapping principle is made with the newly established fixed point theorem of this paper and it is shown in case of a certain *D*-metric, the above two fixed point principles are equivalent.

52

PHENOMENON OF OCCULTATION AND HELIOCENTRIC PLANETARY MODEL IN ARYABHATEEYA-BHASHYA OF NILAKANTHA

K. Ramasubramanian

Dep. of Theoretical Physics, Universtiy of Madras, Guindy Campus, Chennai

Aryabhateeya-bhashya, an elaborate commentary on Aryabhateeya by Nilakantha Somayaji (1444-1550 AD) of Trikkanttiyur, is a very important treatise in the corpus of Indian Astronomy. Here the commentator Nilkantha not only gives a very detailed exposition of the original text which is written in a terse form, but also gives a graphic description of the motion of various celestial objects like the Sun, Moon, Mercury, Venus, etc.

Nilakantha also discusses at length some interesting questions like

1. What is the source of light emitted by the moon and other planets?
2. What is the shape of Moon?
3. Why is it that we see the same face of the Moon?

He also gives a beautiful description of the phenomena of occultation and eclipses. This implies a clear understanding of the latitudinal motion of the planets by the Indian astronomers. Apart from this, he has also given a very clear geometrical model of the planetary motion.

In the lecture we will mainly concentrate on this heliocentric picture described by Nilakantha in his Aryabhateeya-bhashya and other independent works like Siddhanta-darpana. According to this picture all the five planets, Mercury, Venus, Mars, Jupiter and Saturn move around the Mean Sun in eccentric orbits, which in turn moves around the Earth. It may be mentioned that such a clear picture of planetary motion which the Kerala School of astronomers had arrived atleast by 1500 AD, only through naked eye observations, was arrived at by the European traditon much later.

Therefore a detailed and careful study of this text Aryabhateeya-Bhashya by Nilakantha Somayaji, is very much essential, atleast to know what was known to Indian Astronomers by 1500 AD as compared to the other traditions of Astronomy elsewhere in the world.

53
HISTORY OF CRYPTOGRAPHY : THE INDIAN AND ARAB CONNECTIONS

Sundar Lal

Dep. of Mathematics, Institute of Basic Science, Khandari, Agra

54
DIVISION OF YUGA SYSTEM IN THE RATIO OF *4:3:2:1*

Ramesh Chand

Vidya Mandir, Sect. V, BHEL, Hardwar

Indian Savants astronomers and mathematicians divided yuga system in the ratio of *4:3:2:1*. They did not explain the reason of division of yuga system in the ratio of *4:3:2:1*. In this paper an attempt is made to explain the reason of the division of yuga system in the ratio of *4:3:2:1*.

55
NUMBERS IN RĀMĀYAṆA

Dhirendra Joshi and Chitra Kulshrestha*

Army School, Raiwala Cantt. Dehradun
**Govt. Girls Inter College, Rishikesh*

Rāmāyaṇa, One of the oldest epics of indian subcontinent is considered older than Mahābhāratā. Several names for numbers are used while describing the size of Rāma's army in Rāmāyaṇa. The purpose of this paper is to give an account of various numbers used therein.

56
ŚULBAS AND APPROXIMATION TECHNIQUES

Nidhi Handa

Department of Mathematics, Kanya Gurukula Mahavidyalaya, Hardwar

The purpose of this note is to present a few approximation techniques regarding irrational numbers for Katayayana Sulba Sutras.

57
GEOMETRY IN BRĀHMAṆAS

Virendra Arora

Department of Mathematics and Statistics, GKV, Hardwar

The purpose of this note is to discuss construction of altars in Śatapatha Brāhmaṇa and related literature.

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PROFESSOR P. L. SHARMA - MAN AND MATHEMATICIAN

S. K. Gaur and D. Khasdev

J. H. Govt. P. G. College, Betul (M.P.)

The purpose of this paper is present a biography work and contribution of professor P. L. Sharma who supervised about 90 Ph.D Thesis in Mathematics and expired a few years ago.

59

GEOMETRY : A HISTORICAL PERSPECTIVE

Dinesh Singh

University of Delhi, Delhi

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R. C. Gupta

Rasabahar Colony, Jhansi

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AN EMAIL FROM PROFESSOR EMERITUS COONCE OF U.S.A.

December 3, 1999

Principal S. L. Singh
Principal, Science College & Organizing Seceretary
Gurukula Kangri University, Hardwar 249404
FAX : 91-133-416698

Dear Principal Singh,

It with deep regret that I must withdraw from participation in the International Conference on History of Mathematics. For the past few weeks I have had very serious health problems. Upon the advice of my physicians I will be unable to make a journey as long as would be required to attend the conference.

Please accept my $200.00 as a contribution to the conference and the mathematics program in Hardwar. Also, I extend my best wishes for a very successful conference.

Sincerely,

Harry B. Coonce
Professor Emeritus
Department of Mathematics
Minnesota State University
273 Wissink Hall
Mankato, MN 56001
Phone : 507-389-1473 FAX : 507-389-6376

INTERNATIONAL CONFERENCE
ON
HISTORY OF MATHEMATICS

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DAILY PROGRAMME

VENUE : Conference Hall, Faculty of Management

DECEMBER 16, 1999

09:00 REGISTRATION

10:00 INAUGURAL SESSION
Yajna by Acharya Prof. Ved Prakash, ProVC
Welcome Address by Prof. S. L. Singh
Organizing Secretary & Principal, Science College
Inaugural Address by Chief Guest Prof. Sher Singh, Visitor, GKV
Address by Prof. B.S. Yadav, Chairman, Organizing Committee
Address by Padmshree Dr. K.D. Dvivedi
Presidential Address by Dr. Dharm Pal, Vice-Chancellor, GKV
Release of a book on Aryabhata
GKV honours scholars
Vote of thanks by Dr. Virendra Arora, Head, Dep. of Math. & Stat.
National Anthem

11.30 TEA

12:00-12:45	Thomas S. Kuhn & the history and philosophy of mathematics : Prof. SASAKI CHIKRA, University of Tokyo, Japan Chairperson : Prof. B. S. Yadav
12:45-13:30	Aryabhata the father of Hindu Mathematics Mr. S. DASGUPTA of Calcutta Chairperson : Prof. R. C. Gupta
13:30	LUNCH
15:00-15:45	The Gregorian calaender : a scientific masterpiece of late renaissance : Prof. HEINER LICHTENBERG of Germany Chairperson : Prof. Sasaki Chikara
15:45-16:30	Place of Ramanujan in the History of Mathematics : Prof. J.P. AMBASHT, Columbia University, U.S.A. Chairperson : Prof. Paul. Gerdes
16:30	TEA
16:45-17:30	TALKS / PAPER READING SESSION Talks by Dr. K. K. VELUKUTTY AND Dr. B. C. DHAGE Papers by .... Chairperson : Prof. K. S. Choudhary
17:30-19:30	Visit to Har-Ki-Pauri & Dinner

DECEMBER 17, 1999

09:30-10:00	Mathematics in the maghrib XIth-XVIthC- an overview : Prof. A. CHEDDADI, M. V. Univ., Morocco Chairperson : Prof. J. P. Ambasht, USA
10:00-10:30	Andre Weil's India in early thirties : Prof. B. S. YADAV, Delhi Chairperson : Prof. A. Cheddadi
10:30-11:00	Hypergeometric Functions....... - A historical approach : Prof. H. L. MONACHA, Retd. from I.I.T, Delhi Chairperson : Prof. J. M. C. Joshi
11:00-11:15	TEA
11:15-11:45	The Muslim contribution to mathematics : Prof. A. KAPUR, Delhi-Jamia Chairperson : Prof. S. R. Pant, Nepal
11:45-12:15	Some aspects of the cultural history of mathematics : Prof. K. S. CHOUDHARY, Calcutta Chairperson : Prof. A. B. Lohani, Nainital
12:15-:12:45	Some General aspects of Aryabhata : Prof. R. C. GUPTA, Jhansi Chairperson : Padmshree Dr. K. D. Dvivedi
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17:00-19:00	Shri Ganga Aarti, Blessing by Svami Chidanand Sarsvati Muni Ji and Prasadam

DECEMBER 18, 1999

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10:00-10:30	History of mathematics in context of Nepal : Prof. S. R. PANT, Nepal Chairperson : Prof. A. Kapur, Delhi-Jamia
10:30-11:00	A development of special functions... : Prof. J. M. C. JOSHI, Nainital Chairperson : Prof. H. L. Monacha, Delhi
11:00-11:15	TEA
11:15-11:45	A history of evolution of functional analysis Prof. R VASUDEVAN, Delhi Chairperson : Prof. P. K. Jain, Delhi
11:45-12:15	Pre-historic origin of the cover up rule for the partial fraction : Prof. B. P. AMBSHT, Bokaro Chairperson : Prof. P. Jha, Supaul (Bihar)
12:15-12:45	A history of bieberbach conjecture : Prof. P. K. JAIN, Delhi Chairperson : Prof. T. C. Panda, Behrampur
12:45-13:05	History of boundary layer : Prof. T. C. PANDA, Behrampur Chairperson : Prof. G. S. Pandey
13:05-13:20	Limit concept in ancient Indian mathematics : Prof. S. L. SINGH, GKV Chairperson : S. M. Bhave, Pune
13:20-14:15	LUNCH
14:15-14:40	Geometrical treatment of progressive series by Sankara : Prof. V. MADHUKAR MALLAYYA, Trivandrum Chairperson : Prof. Aloke N. Sensharma, Shantiniketan

14:40-15:00	Combinatorial problems in Hindu mathematics : Dr. A. S. Grewal, South Africa Chairperson : Dr. K. K. VELUKUTTY, Coimbatore
15:00-15:20	Geometry in rituals : Dr. VINOD MISHRA, Longowal Chairperson : Dr. Virendra Arora, GKV
15:20-15:40	Veezanka and some concepts in modern algebra : Prof. S. K. BHAKAT, Shriniketan Chairperson : Prof. Anupam Jain, Indore
15:40-16:00	Acharya Sridhara and Mahavira..... : Dr. ANUPAM JAIN, Indore Chairperson : Prof. R. P. Pant, Pantnagar
16:00-16:15	TEA
16:15-17:00	BRIEF TALKS/PAPER READING SESSION Chairperson : Prof. Dinesh Singh, Delhi
17:00-17:45	ROUND TABLE DISCUSSION Chairperson : Prof. J. P. Ambsht **Panelist :** Prof. Sasaki Chikara (Tokyo, Japan) Prof. H. Lichtenberg (Germany) Prof. K. D. Dvivedi (Gyanpur) Prof. P. Gerdes (Mozambique) Prof. S. R. Pant (Nepal) Prof. S. Dasgupta (Calcutta) Prof. B. S. Yadav (Delhi) Prof. A. Kapur (Delhi) Prof. S. L. Singh (Hardwar) Prof. G.S. Pandey (Ujjain) Prof. P. Jha (Bihar)
17:45-18:00	Valedictory function
18:00-19:30	Cultural Programme/A Visit to Shantikunj/Dinner

DECEMBER 19, 1999

07:15-10:30	Visit to Himalayas / New Tehri
10:30-11:00	TEA at Bhagirathipuram, New Tehri
11:00-12:00	Local Visit
12:00-12:30	Lunch at Bhagirathipuram
12:30	Departure from New Tehri
15:30-16:15	A visit to New Madhuban Ashram, Muni-ki-Reti & TEA

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प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका

JOURNAL OF NATURAL & PHYSICAL SCIENCES

गुरुकुल कांगड़ी विश्वविद्यालय, हरिद्वार
Gurukula Kangri Vishwavidyalaya, Hardwar

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DUAL OF MAXIMUM ENTROPY PRINCIPLE : SOME APPLICATIONS

N.C. Das* & S.K. Mazumdar**

(Received 20.11.1997 and after revision 14.11.1998)

ABSTRACT

The paper deals with the study of the rule of the dual of maximum-entropy principle (both constrained and unconstrained) in the derivation of the different distribution laws of quantum statistics. The analogy with some urban systems has been pointed out.

Keywords : Quantum statistics, Maximum-Entropy principle, Dual problems, Urban system.

A.M.S. Classification Number : 82A05

INTRODUCTION

The extremal principles play significant roles in characterising the behaviour of a system. The principles of maximum-entropy and minimum energy exist in dual form of other in classical thermodynamics [1]. The maximum-entropy based on Shannon's informational measure of entropy initiated by Jaynes [4] has increased the importance of these principle not only in thermodynamics and statistical mechanics but also in other branches of Science and Technology [5,10]. The equivalence of these two principles of maximum-entropy and minimum-energy is an important property both in classical and statistical thermodynamics [1, 2, 5]. The object of the present note is to study the interrelation and equivalence of these two principles in the context of both quantum and urban systems.

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This consists of two parts : the first deals with the constrained and the second with the unconstrained dual problems. The analogy between quantum system with some urban systems however rests on the identification of energy with cost or money [10].

MAXIMUM-ENTROPY PRINCIPLE : CONSTRAINED DUAL PRINCIPLE

We consider a system consisting of N molecules classified into n-classes (energy-states). Let $N_i (i=1,2\ldots, n)$ be the occupation number of the ith class. The entropy of the quantum system is given by [8].

$$S = -\sum_{i=1}^{n} N_i \ln N_i - \frac{1}{a}\sum_{i=1}^{n}(1-aN_i)\ln(1-aN_i) \tag{2.1}$$

where $a = 0$ for Maxwell-Boltzmann systems, $a = 1$ for Bose systems and $a = -1$ for Fermi systems. For simplicity, we have assumed the degeneracy of the different classes to be equal to unity. The main problem of the statistical mechanics is to determine the distribution $\{N_i\}$ of the molecules among the different classes subject to the constraints of fixed number of particles and fixed energy :

$$\sum_{i=1}^{n} N_i = N(fixed), \sum_{i=1}^{n} N_i E_i = E(fixed) \tag{2.2}$$

where E_i is the energy of a single molecule belonging to the ith class. In statistical mechanics the distribution laws are determined by some extremal principles. We consider here the principles of maximum-entropy and minimum-energy and study the equivalence of these two principles in the context of quantum systems. From the point of view of optimization theory, we consider the estimation of distribution law by maximum-entropy

principle as the primal problem and that by the minimum energy principle as the dual probelm. We thus set the primal and dual problems for the system under consideration as follows :

The primal problem consists of the determinationof the distribution laws of molecules by the maximization of the entropy (2.1) subject to the constrainsts (2.2). The dual form of this problem consists of minimization of the total energy $E=\sum_{i=1}^{n} N_i E_i$ subject to the constraints of fixed values of the number of particles and entropy S given by (2.2) and (2.1) respectively. Both the optimizations lead to the same distribution law.

$$N_i = \frac{1}{e^{\alpha+\beta E_i} + a} \tag{2.3}$$

where the parameters α and β are given by [8]

$$\alpha = -\frac{\mu}{KT}, \ \beta = \frac{1}{KT} \tag{2.4}$$

T being the absolute temperature and μ is the chemical potential.

Let us now study the importance of the dual principle in the characterization of entropy. We assume that the distribution of N_i given by (2.3) is known and that it has been obtained from the principle of minimum energy. We do not know the functional form of the entropy. it is required to determine the expression of entropy of the quantum system. We take the entropy of the system as

$$S = \sum_{i=1}^{n} f(N_i)$$

where the functional of $f(N_i)$ is to be determined. We assume that the distribution (2.3) has been obtained from the minimization of the energy

$E = \sum_{i=1}^{n} N_i E_i$ subject to constraints :

$$\sum_{i=1}^{n} f(N_i) = S \tag{2.6}$$

and $$\sum_{i=1}^{n} f(N_i) = N \tag{2.7}$$

Minimization of $E = \sum_{i=1}^{n} N_i E_i$ subject to (2.6) and (2.7) leads to the relation

$$E_i - \lambda_0 f'(N_i) - \bar{\lambda}_1 = 0$$

or, $$f'(N_i) = \frac{1}{\lambda_0}(E_i - \bar{\lambda}_1) = \alpha_0 + \alpha_i E_i \tag{2.8}$$

where $$\alpha_0 = -\frac{\bar{\lambda}_1}{\lambda_0} \text{ and } \alpha_1 = \frac{1}{\lambda_0} \tag{2.9}$$

For N_i given by distribution (2.3), the expression (2.8) reduces to

$$f'(x_i) = \alpha_0 + \alpha_1 E_i \tag{2.10}$$

where $$\frac{1}{x_i} = \exp(\alpha + \beta E_i) + a$$

or, $$E_i = \frac{1}{\beta} \ [\ln (\frac{1}{x_i}-a)-\alpha] \tag{2.11}$$

So, we have

$$\begin{aligned} f'(x_i) &= \alpha_0 + \frac{\alpha_1}{\beta}[\ln (\frac{1}{x_i}-a)-\alpha] \\ &= A+B \ \ln \ (\frac{1}{x_i}-a) \\ &= A+b \ \ln \ (1-ax_i) - b \ \ln \ x_i \end{aligned} \tag{2.12}$$

Integrating (2.12) we get, $f(x_i) = -Bx_i \ln \ x_i - \frac{B}{a}(1-ax_i) \ln \ (1-ax_i) + Ax_i + V(a)$

So the entropy of the system reduces to the form

$$S = - B \sum_{i=1}^{n} N_i \ln N_i - (\frac{B}{a}) \sum_{i=1}^{n}(1-aN_i) \ln (1-aN_i) + nV(a) \tag{2.13}$$

The constant B will depend on the unit of measurement of entropy and can be taken to be the Boltzmann constant. So the entropy of the quantum system reduces to the form

$$S = - K \sum_{i=1}^{n} N_i \ln N_i - (\frac{K}{a}) \sum_{i=1}^{n}(1-aN_i) \ln (1-aN_i) \tag{2.14}$$

where the constant $V(a)$ has been left out without affecting the system.

MAXIMUM-ENTROPY PRINCIPLE : UNCONSTRAINED DUAL PRINCIPEL

In the preceeding section we have considered the dual problem with constraints namely the minimization of the total energy (objective function) subject to the constraints of fixed values of entropy and total number of molecules. In the present section we shall consider dual problem with

unconstrained minimization of an objective function say $Z(\beta)$. The main problem is then to construct an appropriate function sucht that

$$\text{Max} \quad (s(N_1, N_2, \ldots, N_n) = \underset{\beta}{Min}\, Z(\beta) \tag{3.1}$$

$$\sum_{i=1}^{n} N_i E_i = E, \quad \sum_{i=1}^{n} N_i = N$$

This type of problems was considered by Kapur [5,6] for various measures of entropy or information. He, however, neither showed how he obtained the objective function nor did he justified them from physical or any other means. For examples, we consider Kapur's entropy

$$I(f) = -\int_a^b f(x) \ln f(x) dx + \frac{1}{c} \int_a^b (1+cf(x)) \ln (1+cf(x)) dx \tag{3.2}$$

which is nothing but Bose-Einstein and Fermi entropy for quantum gas, the distribution number $\{N_i\}$ being replaced by continuous distribution $f(x)$. The constraints considered by Kapur are

$$\int_a^b f(x) gr(x) dx = ar (r = 1, 2, \ldots m) \tag{3.2}$$

The objective function for dual problem considered by Kapur [5,6] is

$$z(\beta) = -\sum_{i=1}^{n} \lambda_r a_r - \frac{1}{c} \int_a^b \ln\left(e^{-\sum \beta_r g_r(x)} - c\right) dx + \frac{1}{c} \int_a^b \sum_{i=1}^{n} \beta_r g_r(x) dx \tag{3.4}$$

For quantum gas the first two terms of (3.4) combined corresponds to the thermodynamic potential as we shall see but what is the physical significance of the third term of the r.h.s. of (3.4) ? In

fact, third term is without any physical significance. We shall show that the first two terms combined together is the expression of thermodynamic potential and it serves as the required objective function for the dual problem under consideration.

The whole system of quantum gas can be considered as an aggregate of n-subsystems consisting of N_i molecules each having energy E_i ($i = 1, 2, ..., n$). The energy of the ith subsystem is $N_i E_i$. The partition function for the whole system can then be written as [9].

$$z(\beta) = \sum \exp\left(-\beta \sum_{i=1}^{n} \right) N_i E_i$$

where summation is taken over

$$z(\beta) = \sum \exp\left(-\beta \sum_{i=1}^{n} \right) N_i E_i \tag{3.5}$$

$$\{N_i, N_2, ..., N_n, \sum_{i=1}^{n} N_i = N\}$$

The summation under constraints in (3.5) can be reduced into a summation without any constraint or restriction

by the relation $\ln z(\beta) = \alpha N + \ln \bar{z}(\beta)$ (3.6)

where the function $\bar{z}(\beta)$, called grand partition function is given by

$$\bar{z}(\beta) = \sum_{\{N_i, N_2, ..., N_n\}} e^{-\lambda \sum_{i=1}^{n} E_i N_i} \tag{3.7}$$

where the sum (3.7) is over all possible numbers $\{N_i, N_2, ..., N_n\}$ without any restriction $\bar{z}(\beta)$ can be evaluated easily as follows

$$\bar{z}(\beta) = \sum_{\{N_1, N_2, ..., N_r\}} e^{-(\alpha+\beta E_1)N_1 - (a+bE_2)N_2 - ....} \tag{3.8}$$

If we consider Bose gas, N_i vary from 0 to ∞ then) then $\bar{z}(\beta)$ becomes

$$\bar{z}(\beta) = \left(\sum_{N_1=0}^{n} e^{-(\alpha+\beta E_1)}\right)\left(\sum_{N_2=0}^{n} e^{-(\alpha+\beta E_2)}\right).....$$

$$= \frac{1}{\left[1-e^{-(\alpha+\beta E_1)}\right]} \mathrm{X} \frac{1}{\left[1-e^{-(\alpha+\beta E_2)}\right]} \mathrm{X}.....$$

$$\text{or } \ln \bar{z}(\beta) = -\sum_r \ln\left[1-e^{-(\alpha+\beta E_1)}\right] \tag{3.9}$$

Then the expression (3.6) gives

$$\bar{z}(\beta) = \alpha N - \sum_r \ln\left[1-e^{-(\alpha+\beta E_1)}\right]$$

$$= \alpha \sum_r N_r - \sum_r \ln\left(1 - e^{-\alpha-\beta E_r}\right) \tag{3.10}$$

The function $\ln z(\beta)$ serves as the required objective function for the unconstrained dual problem. For its minimization with respect to the parameter α leads to

$$d \ln z / d\alpha = 0 \tag{3.11}$$

giving

$$\sum_r N_r - \sum_r \frac{1}{e^{-\alpha+\beta E_{r-1}}} = 0$$

$$\text{or} \quad N_r = \frac{1}{e^{-\alpha+\beta E_{r-1}}} \tag{3.12}$$

which is the Bose-Einstein distribution for quantum gas. In a similar way we can find the distribution of Fermi-Dirac particles. We thus note that

the minimization of the objective function In $\bar{z}(\beta)$ (which is in fact proportional to thermodynamic potential without any restriction leads to the quantum distribution laws. This is nothing but the solution of the unconstraint dual peoblem.

CONCLUSION

In conclusion we note that the above two equivalent principles and distribution laws are of great importance not only in Physics but also in some other branches of social sciences; particularly in the modelling of urban and regional system [10]. Many problem of work-trips, commodity distribution and transportation follow the different quantum distribution and laws. The close analogy between the two apparently different systems was stressed by many authors[3,5,7].

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GEOMETRY IN VEDIC TRADITIONS

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ABSTRACT

In this paper an attempt is made to present a historical development aspect of geometry in *Vedic* Tradition.

INTRODUCTION

According to 'Webesters New International Dictionary' (see [31, p.1049]): "Geometry stands for measurement of the earth's surface. It is the branch of mathematics which investigates the relations, properties and measurement of solids, surfaces, lines and angles, the science that treats of the properties and relations of spatial magnitudes, the theory of space and figures in space."

In India, geometry had its origin in religious rituals. The geometry of the *Vedic* period was the geometry of *Vedas,* the shapes and sizes of altars and *Havankunds.*

According to *Patañjali* (150 B.C.), the commentator of *Paṇini's* Grammar, that there were as many as 1131 or 1137 different schools of the Veda (see [4, p.1]) :

एकविंशतिधा वा ह्वा चयम् एकशतमध्वर्य्यशाखाः।
सहस्त्रवर्त्मा सामवेदः, नवधा आथर्व्वणोवेदः पंचदशभेदो वा।।

Which means :

"There were 21 different schools of the *Ṛgveda,* 101 schools of the *Yajur-veda,* 1000 of the *Sāma-veda,* and 9 or 15 of the *Atharva-veda."*

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KINDS OF GEOMETRY

There are following types of geometry (see [31. p. 1049]) :

1. Algebraic geometry
2. Analytical geometry
3. Circle geometry
4. Denumerative geometry
5. Descriptive geometry
6. Euclidean geometry
7. Geodesic geometry
8. Hyperbolic geometry
9. Intutional geometry
10. Inverse geometry
11. Line geometry
12. Meta geometry
13. Natural geometry
14. Projective geometry
15. Riemannian geometry
16. Speculative geometry

DEVELOPMENT OF GEOMETRY IN *VEDIC* TRADITIONS

1. THE *BHĀHMAṆA* PERIOD (3000 B.C.-1500 B.C.)

If we accept 3000 B.C.as a convenient date for the *Ṛgvedic* culture, the *Aitarey Brāhmaṇa* will have to be assigned a date 2500 B.C., the 'Ś*atapatha Brāhmaṇa* 1500 B.C., the *Tittiriya Samhita* 1600 B.C. and this period is then followed by the period of the *'Śrauta Sūtras'*. It appears that the first real foundations of measurations and also of geometry were laid during the period of the *Brāhmaṇa*. The main *Brāhmaṇas* are :

(1) The *Aitareya Brāhmaṇa,* belonging to the *Ṛgveda school*

(2) Ś*atpath Brāhmaṇa,* belonging to the Ś*ukla Yajurveda* school

(3) The *Tajttiriya Brāhmaṇa,* belonging to the *Kṛṣna Yajurveda* school, the *Kṛṣna Yajurveda* is also known as the *Taittirya Saṁhita.*

(4) The *Gopatha Brāhmaṇa,* belonging to the *Atharvaveda* school.

(5) The *Sāma Brāhmaṇa,* belonging to the *Sāmveda school* (see for instance [29]).

2. ŚULBASŪTRA PERIOD (B.C. 1000 TO 400 A.D.)

The immortal poet *Kālidasa* (flourshied in the 4th century A.D. according to most authorities, in the 3rd century A.D. according to Weber and Lassen, and in the 1st century B.C. (see [19, p. 295]) admired the *śulbavidyā* in the following verse (see [4]) :

क्व शुल्वविवृतविद्या क्व चाल्पवषिया मति।
तितीर्षुरूडुपेनापि दुस्तरमस्मि सागरम्।।

Which means :

How great is the science which revealed itself in the *Śulba,* and how meagre is my intellect ! I have aspired to cross the unconquerable ocean in a mere raft.

Mr. A.C. Burnell was the first to pay attention to the importance of *Śulbasūtras* and remarks in his collection of a *Sanskrit* manuscripts p.29 "We must look to the *Śulba* portions of the *Kalpasūtras* for the earliest beginings of geometry among the *Brāhmaṇs."*

In the sequence of development of the science of mathematics in India, *Śulbasūtra* period is very important. As it is evident from the name, *Śulbasūtra* means "Rule of cords, which is another name of geometry. These *Śūtras* or rules were formulated between (800 B.C.-200 B.C.). There main aim was religious constructions of sacrificial altars called *Vedis*. With their help we learnt rules for constructing geometicals figures such as squares, rectangles, parallelogram, trapezium etc. The rules given for the construction of equivalent rectangles, squares, circles, contains the knowledge of many theorems given in the first, second and sixth

books of *Euclid*. In India *al-Biruni* tried to introduce *Euclid's* Elements through *Sanskrit* during the 11th century. The *Sanskrit* version was prepared by Jagnnath in the 18th century as *Rekhagaṇita* (see [28]). Now-a-days only seven *Śulbasūtras* are known at present : *Baudhāyana*, *Āpastamba*, *Kātyayana*, *Mānava*, *Maitrayana*, *Varāha* and *Vadhula*. These manuals are found separately (see [1], [3], [4] [14], [18], [19], [23], [25], [26], [27] and [29]). The *Śulbasūtras* of *Baudhāyana*, *Āpastamba*, *Mānava*, *Maitrayaṇa*, and *Varāha* belong to the *kṛṣna Yajur-veda* and the *Kātyayana Śulbasūtra to the Śukla-Yajur-veda* (see [4, p.2]).

All the geometrical operations were carried out with the help of measuring Cord or *Śulbi*. The word rajju or *Śulba* means a rope or cord which was used as measuring tape as well as a device for the construction of geometrical figures. The geometrs were called *Śulbavidās* or *Śulbakārs* and geometry was called *Śulbasūtra* (see [1], [3], [4], [23], [26] and [27].)

In the *Pāli* literature, we find the terms *rajjuka* and *rajju-grahka* (rope-holder) for the kings land surveyor. These terms appears in the inscriptions of the *Emperor Ashok* (250 B.C.). In the later works of *Kautilya* (298 B.C.) mentioned a measure of *rajju* which was nearly 60 feet (see [32 p.10]). In the script of *Ashoka* (250 B.C.), the words *rajjuka* and *lajjuka* are identical because in *Sanskrit* grammar the letter 'r' can be used in place of 'l'.

Professor G. Thibaut not only introduced to English reading community the *Baudhāyana Śulbasūtra*, he probably wrote for the first time is a European language a critical paper on the Geometry in the *Śulba* literature under the caption *ON THE ŚULBASŪTRAS* which appeared in the journal of The Asiatic Society of Bengal XI, IV 3 (1875) p. 237-275 (see [18] and [28]).

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SOME RESULTS ON COMMON FIXED POINT II

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ABSTRACT

In this paper some results concerning the existence of a unique common fixed point of the four selfmaps of a complete D-metric space satisfying certain general contractive conditions are proved under the weak commutativity condition and without the requirement of continuity of any one of the four maps as opposed to the case of ordinary metric spaces. Our results generalize the results of Dhage [4] under weaker conditions.

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Key-Words & phrases : D-metric space, contraction map, fixed point, etc.

INTRODUCTION

Motivated by the measures of nearness between two or more objects with respect to a specific properties or characterstic, called the parameter of the nearness, the present author in 1984 in his Ph.D. thesis introduced a new concept of a D-metric space thereby which it has been possible to determine the geometrical nearness, i.e. the distance between two or more points of the set under consideration. It is also now clear that the theory of D-metric spaces has become useful in the study of approximation theory and allied areas of mathematics. A non-empty set X together with a real-valued function $\rho: XxXxX \to [0,\infty)$ is called a D-metric space, denoted by (X,ρ), if D-metric ρ satisfies

(i) $\rho(x,y,z) = 0 \Leftrightarrow x = y = z,$ [coincidence]

(ii) $\rho(x,y,z) = \rho(p\{x,y,z\}),$ [symmetry]

where p is a permutation of x,y,z and

(iii) $\rho(x,y,z) \leq \rho(x,y,a)+\rho(x,a,z)+\rho(a,y,z)$

for all $x,y,z,a \in X$ [tetrahedral inequality]

It is known that D-metric ρ is a continuous function on X^3 in the topology of D-metric convegence which is Hausdorff. See Dhage [2]. Some details along with some standard examples of a D-metric spaces appear in [2].

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In a recent paper [4] the present author proved some common fixed point theorems for three selfmaps of a D-metric space satisfying certain general contractive type condition. In this paper we extend these results to four selfmaps of a D-metric space under the weak commutativity condition and without the requirement of the continuity of any map as opposed to the case of ordinary metric spaces.

PRELIMINARIES

A sequence $\{x_n\} \subset X$ is said to be convergent to point $x \in X$ if $\lim_{m,n} \rho(x_m, x_n, x) = 0$. A sequence $\{x_n\} \subset X$ is called D-Cauchy if $\lim_{m,n,r} \rho(x_m, x_n, x_r) = 0$. A complete D-metric space is one in which every D-Cauchy sequence convreges to a point in it. A mapping $f : X \to X$ is said to be continous if and only if $\lim_{m,n} \rho(fx_m, fx_n, fx) = 0$ whenever $\lim_{m,n} \rho(x_m, x_n, x) = 0$. Finally a subset S of a D-metric space is called bounded if there exists a constant $K>0$ such that $\rho(x,y,z) \leq K$ for all $x,y,z \in S$. and the constant k is called a D-bound of S.

The following lemma is crucial in the sequel.

Lemma 2.1 : Let $\{Yn\} \subset X$ be a bounded sequence with D-bound K satisfying

$$\rho(y_n, y_{n+1}, y_m) \leq \phi^n (K) \qquad \ldots(2.1)$$

for all $m > n \in N$, where

$\phi : R^+ \to R^+$ satisfies $\sum_{n=1}^{\infty} \phi^n (t) < \infty$ for each $t \in R^+$.

Then $\{Yn\}$ is D-Cauchy.

Proof : The proof is given in Dhage [4].

In the following section we prove the main results of this paper.

MAIN RESULTS

Two selfmaps $f, g : X \to X$ are called commuting if $(fg)(x) = (gf)(x)$ for all $x \in X$ and coincidentally commuting if they commute at coincidence points.

It is clear that every commuting pairs of maps is coincidentally commuting, but the converse may not be true. See Dhage [6].

Let $h, k : X \to X, f : x \to k(X)$ and $g : x \to h(X)$ be four mappings. Then by an orbit of four mapping f, g, h and k at a point $x \in X$ is a set $O(f/k, g/h : x)$ in X given by $xO(f/k, g/h : x) = \{x_0\} \cup \{y_n | x_0 = x,\ y_0 = hx_0, y_{2n+1} = fx_{2n} = kx_{2n+1},$

$$y_{2n+2} = gx_{2n+1} = hx_{2n+2},\ n \geq 0\}$$

for some sequence $\{x_n\}$ in X.

By $\overline{O(f/k, g/h : x)}$ we denote the closure of the orbit $O(f/k, g/h{:}x)$ in X.

Let Φ denote the class of all functions $\phi : R^+ \to R^+$ satisfing

(i) ϕ is continuous,

(ii) ϕ is nondecreasing,

(iii) $\phi(t) < t$ for each $t > 0$, and

(iv) $\sum_{n=1}^{\infty} \phi^n(t) < \infty$ for each $t \in R^+$.

We need the following lemma in the sequel.

Lemma 2.1 [4] : If $\phi \in \Phi$, then $lim_n\ \phi^n(t) = 0$ for each $t \in R^+$ and $\phi^n(0) = 0$ for each $n \in N$.

Theorem 3.1 : Let f, g, h and k be four selfmaps of a D-metric space X satisfying

$$\rho(fx, gy, z) \leq \phi(max\ \{\rho(hx, ky, z),\ \rho(hx, fx, z),\ \rho(ky, gy, z)$$
$$1/6\ [\rho(hx, gy, z) + \rho(ky, fx, z)]\}) \qquad .....(3.1)$$

for all $x, y \in X$ and $z \in \overline{O(f/k, g/h, x) \cup O(g/h, f/k, y)}$, where $\phi \in \Phi$.

Further suppose that

(i) $f(X) \subseteq k(X)$, $g(X) \subseteq h(X)$,

(ii) $f(X) \cup g(X)$ is bounded and

$h(X)$ and $k(X)$ are complete, and

(iii) $\{f,h\}$ and $\{g,k\}$ are coincidentally commuting.

Then f,g,h and k have a unique common fixed point.

Proof : Let $x \in X$ be arbitrary and consider the sequence $\{Y_n\} \subset X$ defined by

$$\left.\begin{aligned} & Y_0 = hx_0,\ x_0 = x, \\ & Y_{2n+1} = fx_{2n} = kx_{2n+1} \\ \text{and}\quad & Y_{2n+2} = gx_{2n+1} = hx_{2n+2} \end{aligned}\right] \qquad \text{---(3.2)}$$

for $n = 0, 1, 2, \ldots,$ for some sequence $\{x_n)$ in X.

In view of the hypothesis (i), the sequence $\{y_n\}$ is well defined and $\{y_n\} \subseteq h(y) \cup k(X)$. From the hypothesis (ii), it follows that $\{y_n)$ is bounded, say by a D-bound K.

Now there are two cases. :

Case I : Suppose that $y_n = y_{n+1}$. For some $n \in N$ and let n be even then there exists an $r \in N$ such that $y_{2r} = y_{2r+1}$. By the definition of $\{y_n\}$, we obtain

$$y_{2r} = hx_{2r} = fx_{2r} = hx_{2r+1} = y_{2r+1} \qquad \ldots(3.3)$$

First we show that $y_{2r+1} = y_{2r+2}$. If not, then by (3.1) and (3.3), we get

$$\rho(y_{2r+2}, y_{2r+1}, y_{2r})$$
$$= \rho(fx_{2r}, gx_{2r+1}, y_{2r}) \leq \phi(max\{\rho(hx_{2r}, kx_{2r+1}, y_{2r}), \rho(hx_{2r}, fx_{2r}, y_{2r}),$$
$$\rho(kx_{2r+1}, gx_{2r+1}, y_{2r}),\ 1/6\,[\rho(hx_{2r}, gx_{2r+2}, y_{2r}) + \rho(hx_{2n+1}, fx_{2n}, y_{2r})]\})$$
$$= \phi(\rho(y_{2r+1}, y_{2r+2}, y_{2r}))$$

Which is a contradiction. Hence $y_{2r+1} = y_{2r+2}$. By the definition of $\{Y_n\}$ which implies that

$$y_{2r+1} = kx_{2r+1} = gx_{2r+1} = hx_{2r+2} = y_{2r+2}\,. \qquad \ldots(3.4)$$

We show that $u = y_{2r}$ is a common fixed point of f, g, h and k.

Now

$$\left.\begin{aligned} hw &= hfx_{2r} = fhx_{2r} = fw \\ \text{and} \quad kw &= kgx_{2r+1} = gkx_{2r+1} = gw. \end{aligned}\right] \qquad \ldots(3.5)$$

Again

$\rho(fw, gw, w)$

$\leq \phi(max\ \{\rho(hw, kw, w), \rho(hw, fw, w), \rho(kw, gw, w),$

$\quad 1/6\ [\rho(hw, gw, w) + \rho(kw, fw, w)]\})$

$= \phi(\rho(fw, gw, w))$

which is possible only when $fw = gw = w$, since $\phi \in \Phi$. Therefore we have

$$hw = fw = gw = kw = w.$$

Case II : Assume that $y_n \neq y_{n+1}$ for each $n \in N$. We show that $\{y_n\}$ is D-Cauchy.

Now for $m \geq 2$,.by (3.1) one has

$\rho(y_1, y_2, y_m) = \rho(fx_0, gx_1, y_m)$

$\leq \phi(max\{\rho(hx_0, kx_1, y_m), \rho(hx_0, fx_0, y_m), \rho(kx_1, gx_1, y_m),$

$\quad 1/6\ [\rho(hx_0, gx_1, y_m) + \rho(kx_1, fx_0, y_m)]\})$

$= \phi(max\ \{\rho(y_0, y_1, y_m), 1/6\ [\rho(y_0, y_2, y_m) + \rho(y_1, y_1, y_m)$

$\leq \phi(max\ \{\rho(y_0, y_1, y_m), 1/6\ [\rho(y_1, y_1, y_2) + \rho(y_1, y_2, y_m) + \rho(y_2, y_1, y_m)$

$\quad + \rho(y_0, y_2, y_1) + \rho(y_0, y_1, y_m) + \rho(y_1, y_2, y_m)]\})$

$\leq \phi(max\ \{k, 1/6\ [3K+2\rho(y_1, y_2, y_m)]\})$

$\leq \phi(K).$

Again for m ≥ 3, we get

$$\rho(y_2, y_3, y_m)$$

$$= \rho(fx_2, gx_1, y_m)$$

$$\leq \phi(max\ \{\rho(hx_2, kx_1, y_m), \rho(hx_2, fx_2, y_m), \rho(kx_1, gx_1, y_m),$$

$$1/6\ [\rho(hx_2, gx_1, y_m) + \rho(kx_1, fx_2, y_m)]\})$$

$$= \phi(max\ \{\rho(y_1, y_2, y_m), 1/6\ [\rho(y_2, y_2, y_m) + \rho(y_1, y_3, y_m)]\})$$

$$\leq \phi(max\ \{\phi(K), 1/6\ [3\phi(K) + 2\rho(y_2, y_3, y_m)]\})$$

$$\leq \phi^2(K).$$

In general for any $m \geq n+1$, one has

$\rho(y_n, y_{n+1}, y_m) \leq \phi^n(K)$.

Now an application of Lemma 2.1 yields that $\{y_n\}$ is *D*-Cauchy. Since $h(x)$ and $k(X)$ is complete, there is a point $w \in h(X) \cap k(X)$ such that $lim_n y_n = w$. From (3.2), it follows that

$$w = lim_n y_n = lim_n fx_{2n} = lim_n kx_{2n+1} = lim_n gx_{2n+1} = lim_n hx_{2n+2}.$$

Since $w \in k(X)$, there is a point $v \in X$ such that $w = kv$.

Now for any $n \in N$, one has

$\rho(gv, w, w)$

$$= lim_n\ \rho(fx_{2n}, gv, w)$$

$$\leq lim_n\ \phi(max\ \{\rho(hx_{2n}, kv, w), \rho(hx_{2n}, fx_{2n}, w), \rho(kv, gv, w),$$

$$1/6\ [\rho(hx_{2n}, gv, w) + \rho(kv, fx_{2n}, w)\})$$

i.e. $\rho(gv, w, w) \leq \phi(max \{0, 0, \rho(gv, w, w), 1/6\, \rho(gv, w, w)\})$

$\leq \phi(\rho(gv, w, w))$

and so $gv = w = kv$ since $\phi \in \Phi$.

But $g(X) \subseteq h(X)$, so there is a point $u \in X$ such that

$hu = gv = kv$.

If $fu \neq hu$, then by (3.1), we get

$\rho(fu, gv, w)$

$\leq \phi(max \{\rho(hu, kv, w), \rho(hu, fu, w), \rho(kv, gv, w),$

$1/6[\rho(hu, gv, w)+\rho(kv, fu, w)]\})$

$= \phi(max \{0, \rho(fu, gv, w), 0, 1/6\rho(fu, gu, w)\})$

$= \phi(\rho(fu, gv, w)$

which is a contradiction. Hence

$fu = hu = w = gv = kv$.

By coincidentally commutativity of $\{f, h\}$, it follows that $fhu = hfu$, i.e. $fw = hw$. We show that w is a fixed point of f. If $fw \neq w$, then by (3.1),

$\rho(fz, w, w) = \rho(fw, gv, w)$

$= \phi(max \{\rho(hw, kv, w), \rho(hw, fw, w)\ \rho(kv, gv, w),$

$1/6[\rho(hw, gv, w)+\rho(kv, fw, w)]\})$

$= \phi(max \{\rho(fw, w, w), \rho(fw, fw, w), \rho(w, w, w),$

$1/6[\rho(fw, w, w)+\rho(w, fw, w)]\}$

$$= \phi(max\ \{\rho(fw, w, w), \rho(fw, fw, w)\}).$$

Since $\rho(fw, w, w) \le \phi(\rho(fw, w, w))$ is not possible, we have

$$\rho(fw, w, w) \le \phi(\rho(fw, fw, w). \qquad (*)$$

Again the duplicating the above arguments, we get

$$\rho(fw, fw, w) = \rho(fw, gv, w)$$

$$\le \phi(max\ \{(\rho(fw, w, w), \rho(fw, fw, w)\})$$

$$= \phi(\rho(fw, w, w)). \qquad (*)$$

Substituting (**) in (*), we get

$$\rho(fw, w, w) \le \phi^2\ (\rho(fw, w, w))$$

which is a contradiction and hence $fw = w = hw$.

Similarly if $\{g, k\}$ are coincidentally commuting, then by repeating the above arguments, it is proved that $gw = w = kw$. Thus $fw = hw = gw = kw = w$, i.e. f, g, h and k have a common fixed point.

To Prove uniqueness, let w^* $(\neq w)$ be another common fixed point of f, g, h and k. Then by (3.1),

$$\rho(w, w^*, w)$$

$$= \rho(fw, gw^*, w)$$

$$\le \phi(max\ \{\rho(hw, kw^*, w), \rho(hw, fw, w), \rho(kw^*, gw^*, w),$$

$$1/6\ [\rho(hw, gw^*, w)+\rho(kw^*, fw, w)]\})$$

$$= \phi(max\ \{\rho(w, w^*, w), \rho(w^*, w, w^*)\})$$

$$= \phi(\rho(w^*, w, w^*).$$

$\rho(w^*, w, w^*) \leq \phi(\rho(w, w^*, w))$

The combination of these two statements fields

$\rho(w, w^*, w) \leq \phi^2(\rho(w, w^*, w)$

which is a contradiction. Hence $w = w^*$. This completes the proof.

Corollary 3.1 : Let f, g, h and k be four selfmaps of a D-metric X p, q, r, s, the positive integers, satisfying

$$\rho(f^p x, g^q y, z) \leq \phi(max\ \{\rho(h^r x, k^s y, z), \rho(h^r x, f^p x, z), \rho(k^s y, g^q y, z), 1/6\ [\rho(h^r x, g^q y, z)+\rho(k^s y, f^p x, z)]\}) \quad \ldots(3.6)$$

for all $x, y \in X$ and $\overline{z \in O(f^p / k^s, g^q / h^r; x) \cup O(g^q / h^r, f^q / k^s; y)}$, where $\phi \in \Phi$.

Further suppose that

(i) $f^p(X) \subseteq k^s(X)$ and $g^q(X) \subseteq h^r(X)$,

(ii) $f^p(X) \cup g^q(X)$ is bounded and $h^r(X)$ and $g^q(X)$ are complete.

(iii) f, g, h and k commute pairwise.

Then f, g, h and k have a unique common fixed point.

Proof : By Theorem 3.1, f^p, g^q, h^r, k^s have a unique common fixed point say, $w \in X$, i.e. it is a point such that

$w = f^p w = g^q w = h^r w = k^s w.$

Since h commute with each of f, g and k we have

$hw = hf^p w = f^p(hw),$
$hw = hg^q w = g^q(hw)$ and $hw = hk^s w = k^s(hw).$

This shows that hw is again a common fixed point of f^p, g^q, h^r and

k^s. By uniqueness of w, we get $hw = w$. Similarly it is proved that $fw = gw = kw = w$. The proof is complete.

Next we prove a result concerning the unique common fixed of four maps on a D-metric space satisfying a contractive condition more general than (3.1) under certain compactness type conditions.

Theorem 3.2 : Let f, g, h and k be four selfmaps of a D-metric space X satisfying

$$\rho(fx, gy, z) < max\,\{\rho(hx, ky, z),\ \rho(hx, fx, z),\ \rho(ky, gy, z),$$
$$1/6[\rho(hx, gy, z)+\rho(ky, fx, z)]\} \qquad \ldots(3.7)$$

for all x, y $\in X$ and $z \in \overline{O(f/k, g/h, x) \cup O(g/h, f/k, y)}$,, for which the right hand side is not zero.

Further suppose that

(i) $f(X) \subseteq k(X)$ and $g(X) \subseteq h(X)$,

(ii) $\{f, h\}$ and $\{g, k)$ are coincidentally commuting pairs, and

(iii) h and k are compact, and f, g, h and k are continuous on $h(X) \cup k(X)$.

Then f, g, h and k have a unique common fixed point.

Proof : First we note that if f, g, h and k have a common fixed point, then from (3.7) it follows that the common fixed point is unique.

Define $A = h(X) \cup k(X)$. Obviously $f, g, h, k : A \rightarrow A$. Since h and k are compact map, $h(X)$ and $k(X)$ are compact and so is also A. Therefore both sides of (3.7) are bounded on A.

Now there are two cases :

Case I : Suppose that the right hand side of the inequality (3.7) is zero for some $(x, y, z) \in A^3$. Then $hx = fx = ky = gy = w$ and it can be shown as in the proof of Theorem 3.1 that w is a common fixed point of

f, g, h and k, and so it is unique.

Case II : Suppose that the right hand side of (3.7) is positive for all $x, y, z \in A$. For the sake of convinence, we denote it by $M(x, y, z)$.

Define a function $T : A^3 \to R^+$ by

$$T(x, y, z) = \frac{\rho(fx, gy, z)}{M(x, y, z)} \qquad \ldots(3.8)$$

for $x, y, z \in A$.

Clearly the function T is well defined because $M(x, y, z) \neq 0$ for all $x, y, z \in A$. Since f, g, h and k are continuous, by the compactness of A, it follows that T attains its maximum at some point $(u, v, w) \in A^3$. Call the value c. From (3.7) it is clear that $0<c<1$. Now by the definition of c, $T(x, y, z) \leq c$ for all $x, y, z \in A$. *Hence by (3.8) we get*

$$\begin{aligned}\rho(fx, gy, z) &\leq c\, M(x, y, z)\\ &= c \max \{\rho(hx, ky, z), \rho(hx, fx, z), \rho(ky, gy, z),\\ &\quad 1/6[\rho(hx, gy, z)+\rho(ky, fx, z)]\} \qquad \ldots(3.9)\end{aligned}$$

for all $x, y, z \in A$. *As A* is compact, it is bounded and complete. Now the desired conclusion follows by an application of Theorem 3.1 with $\phi(t) = ct$, $0<c<1$. This completes the proof.

Corollary 3.2 : Let f, g, h and k be four self maps of a D-metric space X, p, q, r, s, the positive integers, satisfying

$$\begin{aligned}\rho(f^p x, g^q y, z) < \max\{&\rho(h^r x, k^s y, z), \rho(h^r x, f^p x, z), \rho(k^s y, g^q y, z),\\ &1/6[\rho(h^r x, g^q y, z)+\rho(k^s y, f^p x, z)]\} \qquad \ldots(3.10)\end{aligned}$$

for all $x, y \in X$ *and* $z \in \overline{O(f^p / k^s, g^q / h^r; x) \cup O(g^q / h^r, f^q / k^s; y)}$, for which the right hand side is not zero.

Further suppose that

(i) $f^p(X) \subseteq k^s(X)$ and $g^q(X) \subseteq h^r(X)$,

(ii) f, g, h and k commute pairwise, and

(iii) h^r and k^s are compact, and f^p, g^q, h^r and k^s are continous on $h^r(X) \cup k^s(X)$.

Then f, g, h and k have a unique common fixed point.

Proof : The proof is similar to Theorem 3.2 and now the conclusion follows by an application of Corollary 3.1. The proof is complete.

When $k = I$ or $h = k = I$, I the identity map on X, our results reduce to the results of [4] under slightly weaker conditions.

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CYLINDRICAL IONIZING SHOCK WAVES WITH RADIATION HEAT FLUX IN UNIFORM ATMOSPHERE

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ABSTRACT

Similarity solutions behind a cylindrical shock wave with radiation heat flux are studied. The ionizing shock is assumed to be propagating in medium at rest with a uniform density permeated by an azimuthal magnetic flux generated by a constant line current passing along with the line source of the blast wave. The electrical conductivity of the gas is infinite behind the shock and zero ahead of it.

INTORDUCTION

The propoagation of ionizing strong shock waves has been studied by Greenspan [2], Christer [1] and Ranga Rao and Ramanna [4] without taking into account radiation effects. Singh [5] has studied the same problem with thermal radiation.

Helliwell [3] has discussed the piston problem in the pressence of radiation heat flux using similarity method.

Recently Singh and Vishwakarma [6], Singh and Singh [8], Singh and Dube [7], and Singh and Singh [9] have investigated their problems in radiative magneto gas dynamics in detail.

In the present paper a similarity solution is developed describing the propagation of a cylindrical shock in an applied azimuthal magnetic field with uniform density generated by constant line current passing along the axis of symmetry. The counter gas pressure and radiation heat flux have taken into account.

A modified gas dynamic shock wave in which electrical conductivity of the gas is infinite behind it and zero ahead of its front, has been taken into

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consideration. The radiation pressure and radiation energy have been neglected. The gas in the undisturbed field is assumed to be at rest.

We have also assumed that the gas to be grey and opaque and the shock to be transparent and isothermal. The total energy of the explosion is constant.

SELF SIMILAR FROMULATION

Cylindrical polar coordinates, where r is the radial distance from the line current which is taken to pass in a positive sense along the axis of symmetry, are used here. The equation of conservation of mass, momentum, energy and magnetic flux in the finite conduction region behind the wave are

$$\frac{d\rho}{dt}+\frac{\rho}{r}\frac{\partial}{\partial u}(ru)=0 \tag{1}$$

$$\frac{du}{dt}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{h}{\rho r}\frac{\partial}{\partial r}(rh)=0 \tag{2}$$

$$\frac{dh}{dt}+h\frac{\partial u}{\partial r}=0 \tag{3}$$

$$\frac{de}{dt}+p\frac{d}{dt}\left(\frac{1}{\rho}\right)+\frac{1}{\rho}\frac{\partial}{\partial r}(rF)=0 \tag{4}$$

Here, ρ is the density, p is the pressure, u is the radial velocity, h is the azimuthal magnetic field, F is the heat flux, t is the time end e is the material energy. The magnetic pemeablility is taken to be unity.

For the ideal gas we have.

$$e=\frac{p}{\gamma-1},\ p=\Gamma\rho T \tag{5}$$

where γ is the adiabatic gas index, T is the temperature and Γ is the gas constant. Assuming local thermodynamic equilibrium and using Plank's diffusion approximation, we have

$$\frac{\partial F}{\partial r} = 4\mu . aT^4 \tag{6}$$

where μ is Plank's mean absorption coefficient a is the Stefan Boltzamann constant, and T the absolute temperature.

We take μ as a power law function of the density and temperature (Wang, 1966) as

$$\mu = \mu_0 \rho^\alpha T^\beta \tag{7}$$

where μ_0, α and β are constants.

The disturbance is headed by an isothermal shock and the conditions are

$$\rho_1(v - u_1) = \rho_0 v = m_s$$

$$p_1 - p_0 = m_s u_1$$

$$e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}(v - u_1)^2 - \frac{F_1}{m_s} = e_0 + \frac{p_0}{\rho_0} + \frac{1}{2}v^2 \tag{8}$$

$$h_1 = h_0$$

$$T_1 = T_0$$

where subscripts 1 and 0 are for the regions just behind and just ahead of the shock surface respectively and denotes the shocks velocity.

For an isothermal shock,

$$\frac{p_0}{\rho_0} = \frac{{a_0}^2}{\gamma} \tag{9}$$

where a_0 is the sound velocity.

SIMILARITY SOLUTIONS

From general dimensional consideration the form of the solutions of equations of motion are,

$$\rho = \rho_0 R(\eta)$$

$$u = \frac{r}{t} U(\eta)$$

$$p = \rho_0 (\frac{r}{t})^2 P(\eta)$$

$$h = (\rho_0)^{1/2} \frac{r}{t} H(\eta) \tag{10}$$

$$F = \rho_0 (\frac{r}{t})^3 Q(\eta)$$

where the quantities R, U, P, H and Q are non dimensional functions of the dimensional variables.

$$\eta = \frac{r}{vt} \tag{11}$$

and V is some characteristic parameter, with the dimensions of a velocity. Lines of constant η are straight lines in the (r,t) plane passing through the origin. In particular, the shock wave and the constant front are two such lines. These fronts therfore expand with constant radial velocity. It is convenient to introduce independent variables instead of r and t, the stream function ψ defined by

$$\frac{\partial \psi}{\partial r} = (\frac{\rho}{\rho_0})r, \ \frac{\partial \psi}{dt} = -(\frac{\rho}{\rho_0})ur \tag{12}$$

and the variable ϕ defined as

$$\phi = \frac{2\psi}{r^2} \tag{13}$$

Physically the value of ϕ corresponding to any (r, t) is the ratio of the mass which is at a time t between the axis and the radius r to the mass initially in the

same volume. The value of ϕ at the shocks, therefore, if $Q_s = 1$ in term of the variable (ψ , ϕ) the non dimensional form are

$$u = vU(\phi)$$

$$\rho = \rho_0 \sigma (\phi)$$

$$p = \rho_0 v^2 P(\phi) \tag{14}$$

$$h = (\rho_0)^{1/2} VH(\phi)$$

$$F = \rho_0 V^3 Q(\phi)$$

Where the quantities σ , U, P, H amd Q are the non deimensional funtions of the single non-dimensional variable Φ. The Jocobian of the transformation defined by equation (20) is readily found to be

$$J = \frac{\partial\ (\psi, \phi)}{\partial\ (r, t)} = -\, 2\sigma U v \Phi \tag{15}$$

Since $J = 0$ the transformation defines a one to one mapping between (r,t) and (ψ, Φ). The differential equation

$$\frac{\partial\ t}{\partial\ \Psi} = -\frac{1}{J} \frac{\partial\ \phi}{\partial\ r} = \frac{1}{\sqrt{2}v} \frac{\sigma - \phi}{\sigma\ \phi^{1/2} U \Psi^{\ 1/2}} \tag{16}$$

is easily intergrated to yield.

$$\Psi = \frac{1}{2} v^2 t^2 \left[\frac{\sigma^2 \phi U^2}{(\sigma - \phi)^2} \right] \tag{17}$$

Finally, it is possible to express the variable η as a function of Φ by combining equations (12) and (17)

$$\eta = \left(\frac{\sigma\ U}{\sigma\ -\ \phi} \right) \tag{18}$$

The total energy E is carried by the wave

$$E = K\left(\frac{1}{2}\sigma\ U^2 + \frac{P}{\gamma - 1} + \frac{H^2}{2}\right) \tag{19}$$

is constant if $K = \pi\ \rho_0 v^2 R^2$ is a constant.

In terms of non-dimensional form the fundamental equations of motion become

$$\left.\begin{aligned}
&\sigma U + 2(\sigma - \phi)\sigma \frac{dU}{d\phi} - 2\phi U \frac{d\sigma}{d\phi} = 0 \\
&H^2 + 2(\sigma - \phi)\frac{dP}{d\phi} - 2\phi\sigma U \frac{dU}{d\phi} + 2(\sigma - \phi)\frac{HdH}{d\phi} = 0 \\
&\phi U \frac{dH}{d\phi} + H(\phi - \sigma)\frac{dU}{d\phi} = 0 \\
&Q - \frac{2\phi U}{\gamma - 1}\left(\frac{dP}{d\phi} - \frac{\gamma P}{\sigma}\frac{d\sigma}{d\phi}\right) + 2(\sigma - \phi)\frac{dQ}{d\phi} = 0 \\
&\frac{dQ}{d\phi} = N\left(\frac{\sigma^{\alpha-\beta-4}}{\sigma}\frac{\beta^{4+\beta}}{\phi}\right)
\end{aligned}\right] \tag{20}$$

$$\alpha = \frac{3}{2}, \quad \beta = -2, \qquad N = \frac{2a\mu_0 K^{1/2}}{\Gamma^{4+\beta}}$$

According to Helliwell [3] the ranges of α and β are given below. The quantities α, β are such that

$$0 \le \alpha \le 2, \quad -5 \le \beta \le 7$$

Also according to similarity method, using equations (10) in equation (4), (6) and (7) we get the last two equations of (20) in non-demensional form for the values of $\alpha = \frac{3}{2}$ and $\beta = -2$ which lies within the above limits.

After simplification the shock relations (8) become

$$
\left.
\begin{aligned}
u_1 &= \left(1 - \frac{1}{\gamma M^2}\right)v \\
\rho_1 &= \gamma M^2 \rho_0 \\
p_1 &= \rho_0 v^2 \\
h_1 &= h_0 \\
F_1 &= \frac{1}{2}\left(\frac{1}{\gamma^2 M^4} - 1\right)\rho_0 v^3
\end{aligned}
\right\} \qquad (21)
$$

where $M^2 = (\frac{v_0^2 \rho_0}{\gamma p_0})$ known as Mach number. Substituting (14) into equation (21) and then taking $\phi = 1$ at the shock front, we find the boundary conditions of the problem are

$$
\left.
\begin{aligned}
U(1) &= \left(1 - \frac{1}{\gamma M^2}\right) \\
P(1) &= 1 \\
\sigma(1) &= \gamma M^2 \\
H(1) &= \frac{1}{M_h} \\
Q(1) &= \frac{1}{2}\left(\frac{1}{\gamma^2 M^4} - 1\right)
\end{aligned}
\right\} \qquad (22)
$$

4. RESULTS AND DISCUSSION :

Form the initial conditions (22) equations (20) have been intergrated numerically for

$\gamma = \frac{7}{5}$; $M_h = 10$; $M = 10$

N= 10, 100; $\alpha = \frac{3}{2}$ *and* $\beta = -2$

The distribution of the flow variables are illustrated through the figures 1 to 5 and tables 1 to 2 behind the shock for N=10 and 100.

The effect of radiatin flux on the flow variables is much important as the non-dimensional parameter changes its values form 10 to 100. We find that the radiation flux is negative in the entire region and decreases as the contact front is approached for N=10, but it decreases very fast within the region between ϕ =1 to 0.90 for the values of M=10 when N=100. This shows that there is more absorptin of radiation rather than emissioin as the expanding gaseous element are more and more heated up by radiation.

When we take M=10 the nature of the flow variables is the same for the N=10 and N=100 but their magnitudes are quite different which can be easily seen through the figures and tables. They are generally of increasing order. We find that the change for N=10 is from shock front towords contact surface, but for N=100 the change in the flow variables is very fast near the shock front. One can eaily see that the magnitude of the density varies in between 3500 to 7559, when N=10.

At M=10 and N=10, 100 the range of magnitude of the density is 140 to 38138 and 140 to 751 respectively. For the case when M=500 and N=10 and 100 the rage of the magnitude of the density varies from 350000 to 346823 and 350000 to 352383 respectively. It can be seen in table 1 that the density if of the decreasing order, but in the table 2 it is of increasing order. In table 1 and 2 the velocity and radiation flux are of increasing order.

When we take M=10 and N=100 the velocity and pressure both are of increasing order, but at N=10 velocity first increases up to ϕ = 0.34 and after that it becomes constant, where as pressure increases up to ϕ = 0.32 after that it decreases.

TABLE -1

M=500, N=10

ϕ	U	H	σ	Q	P
1.00	1.00000	0.10000	350000.00000	-0.50000	1.00000
0.95	1.00000	0.10237	349200.71875	-0.50000	1.00156
0.90	1.00000	0.10492	348401.34375	-0.50000	1.00309
0.85	1.00000	0.10766	347608.46875	-0.50000	1.00485
0.80	1.00000	0.11060	346823.03125	-0.50000	1.00784

TABLE - 2

M=500, N=10

ϕ	U	H	σ	Q	P
1.00	1.00000	0.10000	350000.00000	-0.50000	1.00000
0.99	1.00000	0.10074	350800.31250	-0.50000	1.00353
0.98	1.00000	0.10147	351594.68750	-0.50000	1.00707
0.97	1.00000	0.10223	352383.43750	-0.50000	1.01068

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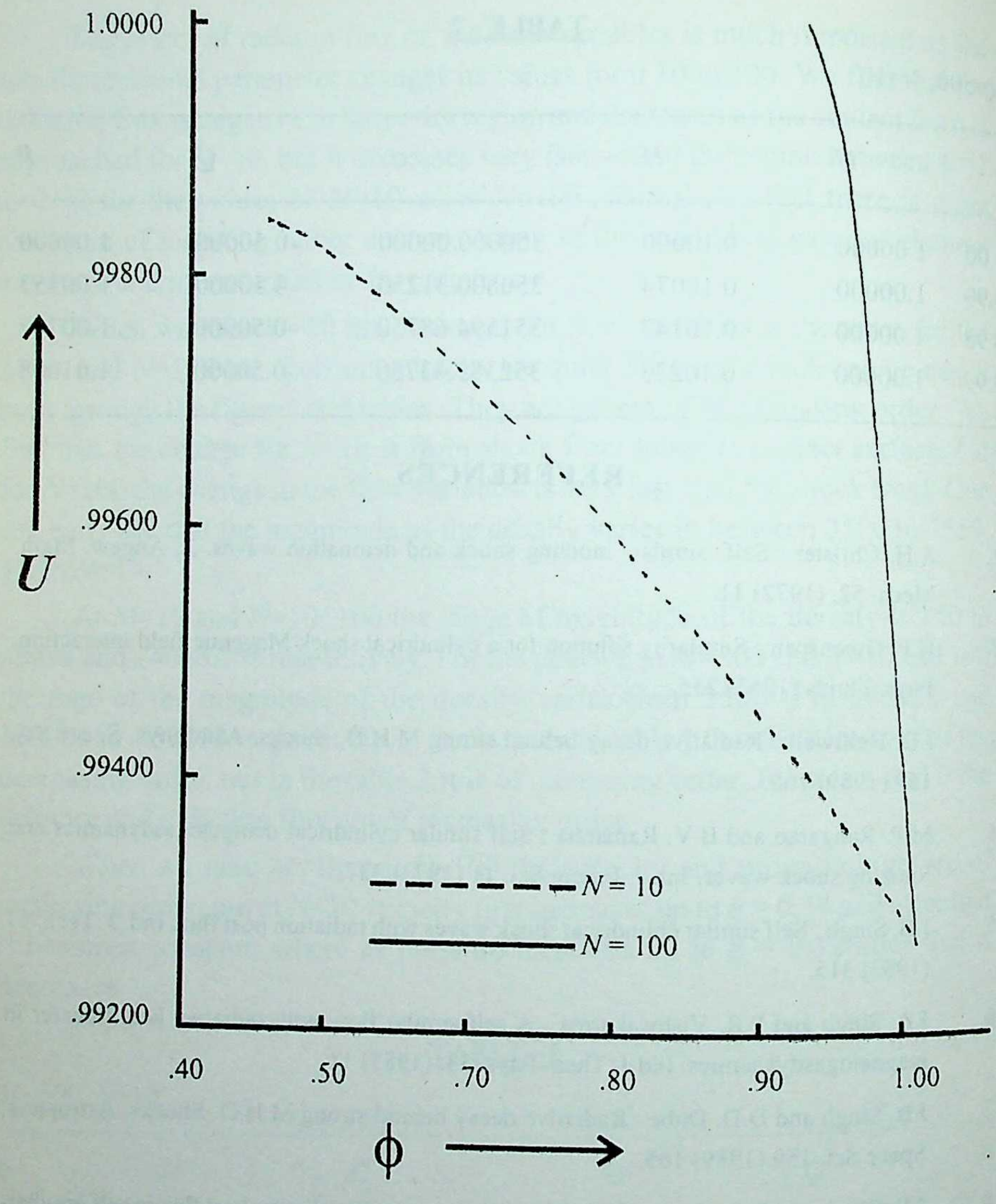

Fig. 1. Velocity distribution for M=10

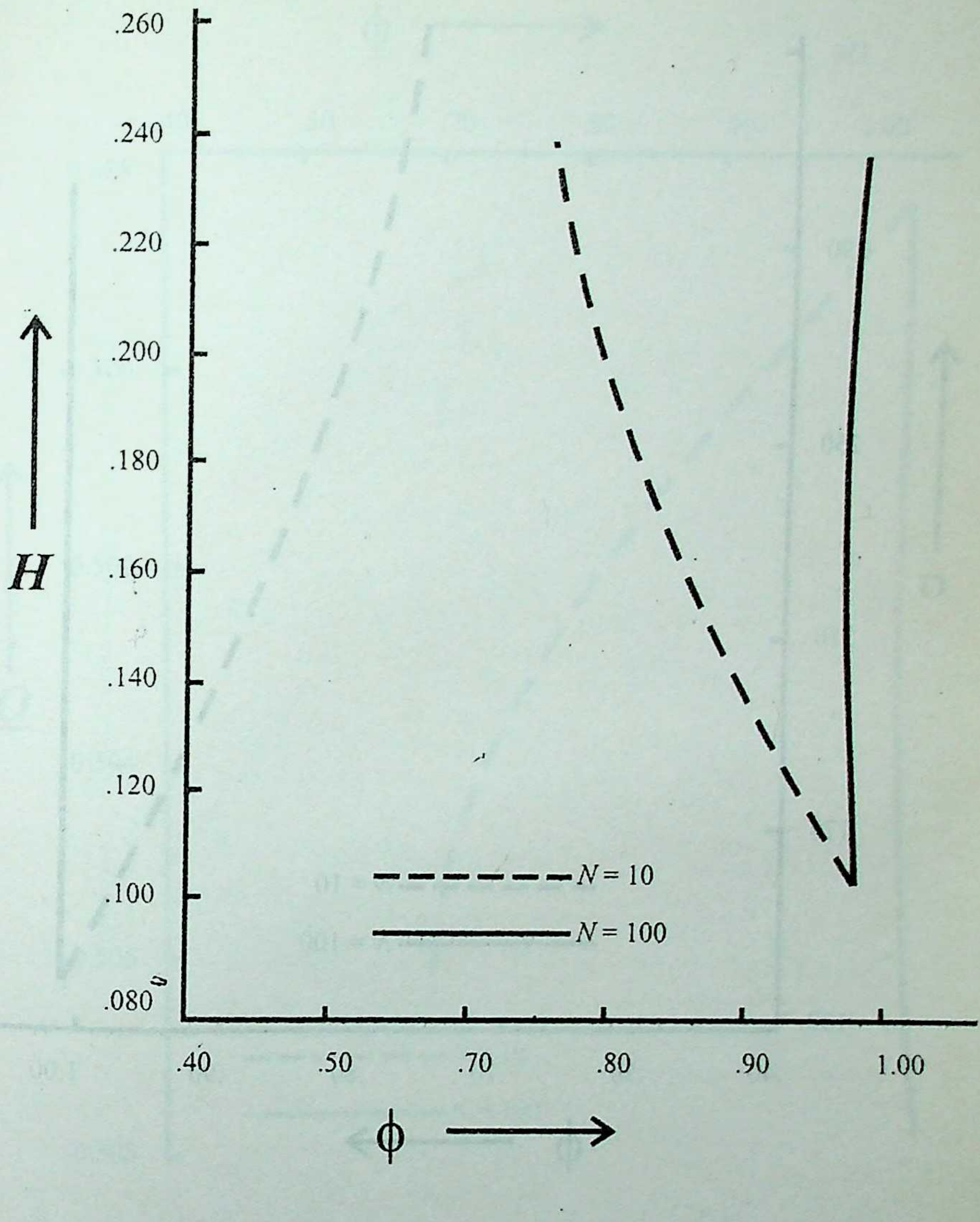

Fig. 2. Magnetic field distribution for M=10

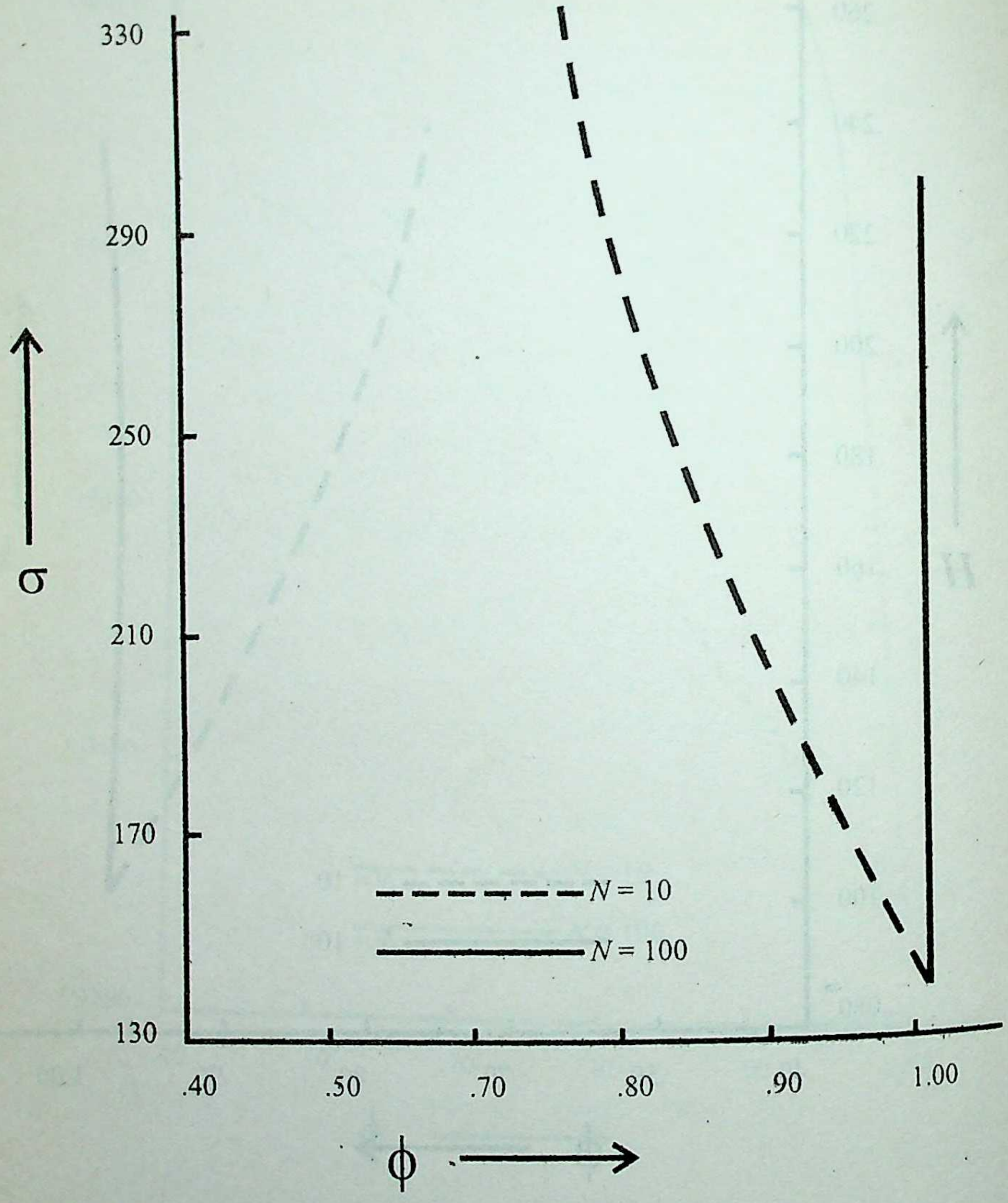

Fig. 3. Density field distribution for M=10

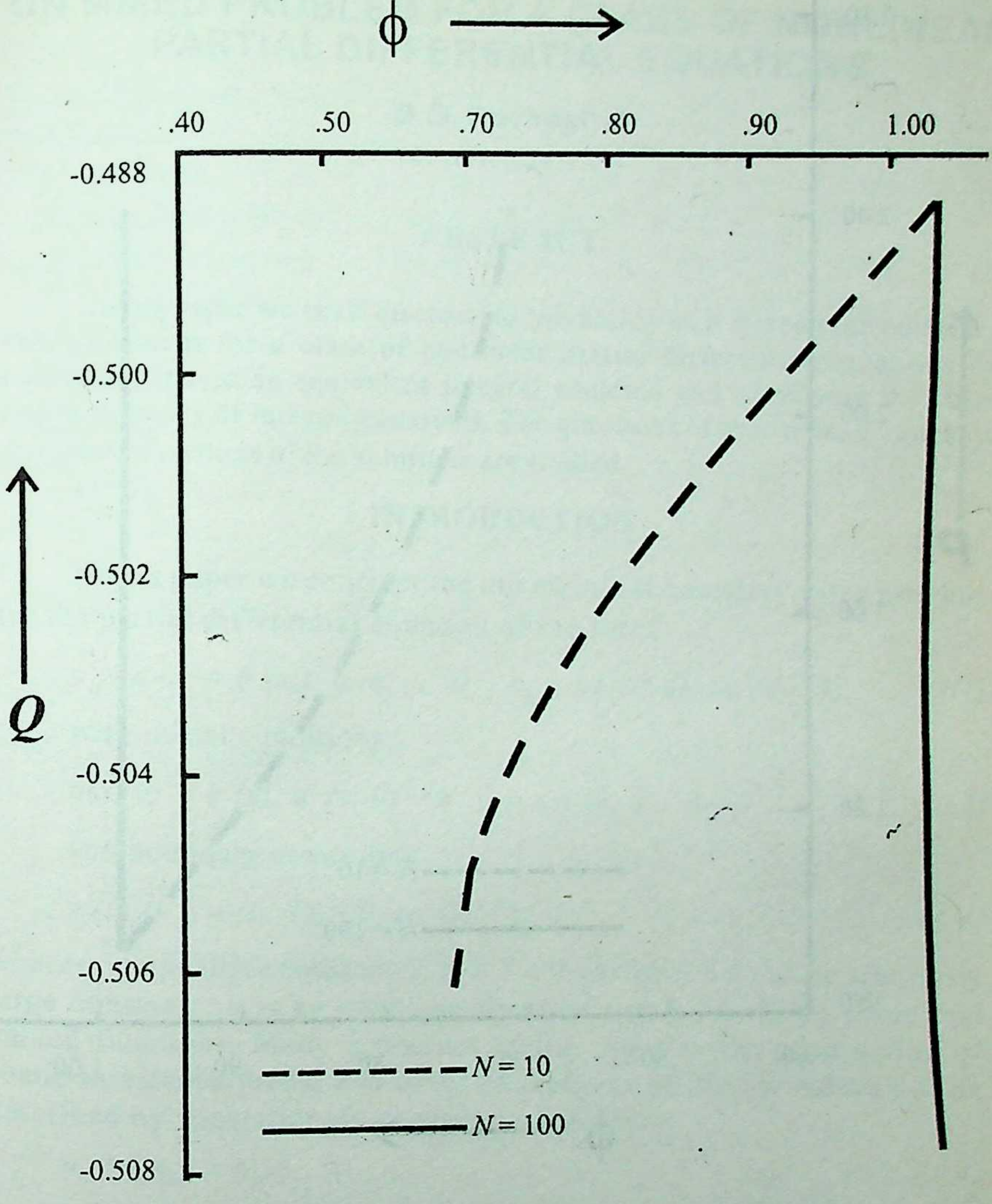

Fig. 4. Radiation flux distribution for M=10

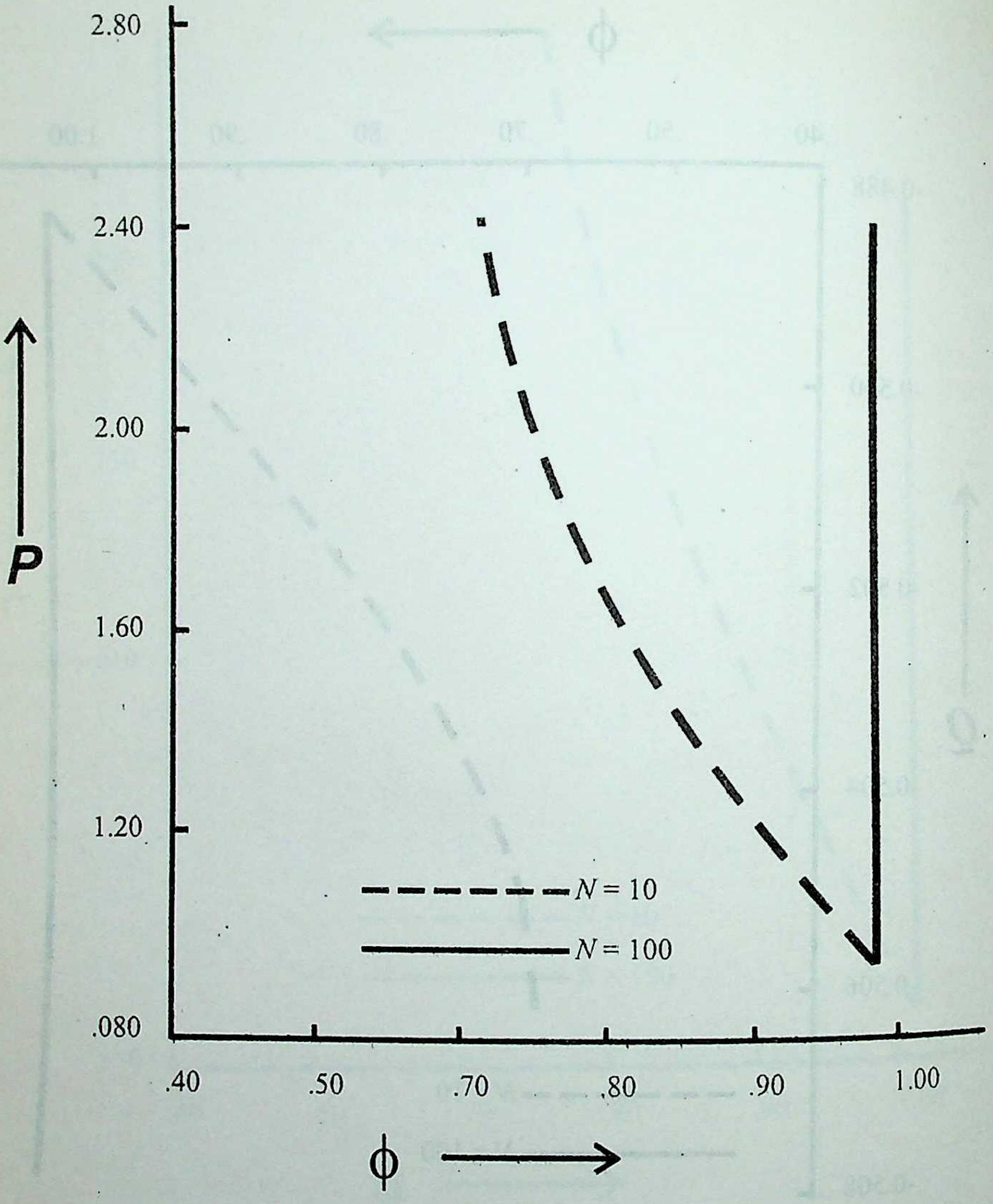

Fig. 5. Pressure field distribution for M=10

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ON MIXED PROBLEM FOR A CLASS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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ABSTRACT

In this paper we shall discuss the solvability of a mixed type boundary value problem for a class of nonlinear partial differential equations by converting it into an equivalent integral equation and employing the basic classical theory of integral equations. The questions of existence, uniqueness and approximations of the solutions are studied.

INTRODUCTION

In this paper we consider the mixed initial boundary value problem for the partial differential equation of the form

$$u_{tt} - au_{xxt} = F(x, t, u, u_x, u_t, u_{tx}, u_{xx}),\ x \in (0, L),\ t \in [0, T], \quad \text{...(1.1)}$$

with initial conditions

$$u(x, 0) = \phi(x),\ u_t(x, 0) = \psi(x),\ x \in (0, L), \quad \text{...(1.2)}$$

and boundary conditions

$$u(0, t) = u(L, T) = 0,\ t \in [0, T], \quad \text{...(1.3)}$$

where a is a positive constant; $L > 0$, $T > 0$ are finite but can be arbitrarily large constants; u is an unknown function and ϕ, ψ, F, are given real valued functions. Many processes taking place in the propagation of sound in viscous media and other phenomena of similar nature can be described by nonstationary equations of the form

$$u_{tt} = au_{xxt} + u_{xx}, \quad \text{...(1.4)}$$

where a is a positive constant and au_{xxt} is a small viscosity. It is well known [7] that (1.4) is neither parabolic nor hyperbolic and hence (1.4) or its general form given in (1.1) is a mixed type problem.

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The problems of existence, uniqueness and other properties of the solutions of various special forms of mixed boundary value problem (MBVP), (1.1) - (1.3) have been studied by many authors by using different techniques, (see [1, 8, 10, 15, 16]). The main purpose of the present paper is to study the existence, uniqueness and approximations of the solutions of (MBVP) (1.1) - (1.3). Our approach is based on converting the problem into an equivalent integral equation by means of the Green's function representation and by using the method introduced by Bielecki [2]. For a detailed account on Bielecki's method in the theory of integral equations we refer the interested readers to the recent survey paper by C.Corduneanu [3]. Further applications of our approach to other mixed type boundary value problems for certain partial differential equations are also given.

STATEMENT OF RESULTS

Let $E=R \times R \times R \times R \times R$ be the product space endowed with the norm $|z|_E = \sum_{i=1}^{5} |z_i|$ for every $z = (z_1, z_2, z_3, z_4, z_5) \in E$, where $R = (-\infty, \infty)$. For any function u(x,t) continuous together with its partial derivatives u_x, u_t, u_{tx}, u_{xx} we denote

$$|u(x,t)|_E = |u(x,t)| + |u_x(x,t)| + |u_t(x,t)| + |u_{tx}(x,t)| + |u_{xx}(x,t)| \qquad \dots(2.1)$$

for $(x,t) \in Q = [0, L] \times [0, T]$. Let S be the space of functions $u(x,t) : Q \to R$ continuous together with the partial derivatives u_x, u_t, u_{tx}, u_{xx} such that

$$|u(x,t)|_E = O\{exp[t+x)]\}, (x, t) \in Q, \qquad \dots(2.2)$$

where λ is a positive constant. In the space S we define the norm (see [2,3,13])

$$|u|_s = \sup_Q \{|u(x,t)|_E exp[-\lambda(t+x)]\} \qquad \dots(2.3)$$

It is easy to see that S with norm defined in (2.3) is a Banach space. We note that the condition (2.2) implies that there exists a constant $K \geq 0$ such that

$$|u(x,t)|_E \leq K exp[\lambda(t+x)], \quad (x,t) \in Q. \qquad \dots(2.4)$$

Using (2.4) in (2.3), we observe that

$$|u|_S \leq K \qquad \ldots(2.5)$$

A function $u(x,t) \in C(Q)$, having a continuous first order t-derivative on $[0, T]$, for all x $\in [0, l]$, whose derivatives occuring in equation (1.1) exist and are continuous for all $(0, L)$ x $(0, T)$, and which satisfies the equation and the initial and boundary conditions (1.2) and (1.3) in the customary classical sence, is called a classical solution of the mixed problem (1.1) - (1.3).

Let $G(x, y, t)$ be the Green's function for the heat equation $w_t = aw_{xx}$ on $(0, L)\ x\ (0, T)$ with the zero Dirichlet boundary data. As in [1, p.203], (see also [6, 9]) we first convert (1.1) - (1.3) into an integral equation by using the Green's function $G(x,y,t)$ such that

$$u_t(x,t) = \int_0^L G(x,y,t)\, \psi(y)dy + \int_0^t\int_0^L G(x,y,t-\tau)\, F(y, \tau, u(y,\tau),$$

$$u_y(y,\tau), u_\tau(y,\tau), u_{\tau y}(y,\tau), u_{yy}(y,\tau) dydt. \qquad \ldots(2.6)$$

Formal integration of (2.6) from 0 to t and the use of the condition $u(x,0) = \phi(x)$, yields the following integral equation

$$u(x,t) = h(x,t) + \int_0^t\int_0^s\int_0^L G(x, y, s-\tau)\, F(y, \tau, u(y, \tau), u_y(y, \tau),$$

$$u_\tau(y,\tau), u_{\tau y}(y,\tau), u_{yy'}(y,\tau))\, dydtds, \qquad \ldots(2.7)$$

where

$$h(x, t) = \phi(x) + \int_0^t\int_0^L G(x,y,s)\, \psi(y)\, dyds. \qquad \ldots(2.8)$$

Thus the solution of the (MBVP) (1.1) - (1.3) is also a solution to the integral equation (2.7) and vice versa, and hence we will deal with the equation (2.7) as an equivalent form of (MBVP) (1.1) - (1.3) in our subsequent discussion.

We use the following notation in order to abbreviate the presentation.

$$N(G, g) = \int_0^t \int_0^s \int_0^L G(x,y,s-\tau)\, g(y, \tau)\, exp\,[\lambda(\tau+y)]\, dyd\tau\, ds, \quad \ldots(2.9)$$

$$N_0(G, g) = \int_0^t \int_0^L G(x, y, t-\tau)\, g(y.\tau)\, exp\,[\lambda(\tau+y)]\, dyd\tau, \quad \ldots(2.10)$$

$$M(G,F(u)) = \int_0^t \int_0^s \int_0^L G(x, y, s-\tau)\, F(y,\tau, u_y(y,\tau), u_\tau(y,\tau), u_{\tau y}(y,\tau),$$
$$u_{yy}(y,\tau))\, dyd\tau\, ds, \quad \ldots(2.11)$$

$$M_0(G, F(u)) = \int_0^t \int_0^L G(x, y, t-\tau)\, F(y,\tau, u(y,\tau), u_y(y,\tau),$$
$$u_\tau(y,\tau), u_{\tau y}(y,\tau), u_{yy}(y,\tau))\, dyd\tau. \quad \ldots(2.12)$$

For convenience we list the following hypotheses used in our further discussion.

(H_1) $\phi(x)$ is twice continuously differentiable on $[0,L]$ and $\phi(0)=\phi(L)=\phi''(0)=\phi''(L)=0$.

(H_2) $\psi(x)$ is continuous on $[0, L]$ and $\psi(0) = \psi(L) = 0$.

(H_3) $F(x,t,r_1,r_2,r_3,r_4,r_5)$ is continuous in $(0,L) \times (0, T) \times R^5$ and is Lipschitz continuous with respect to r_1, $i = 1,\ldots, 5$ uniformly in Q, that is, there exists a nonnegative real valued continuous function $g(x,t)$ defined on Q such that

$$\left|F(x,t,r_1,r_2,r_3,r_4,r_5) - F\left(x,t,\bar{r}_1,\bar{r}_2,\bar{r}_3,\bar{r}_4,\bar{r}_5,\right)\right| \le g(x,t)\sum_{i=1}^{5}\left|r_i - \bar{r}_i\right|, \quad \ldots(2.13)$$

and

$$N(|G|, g) \le \beta_1\, exp\,[\lambda(t+x)] \quad \ldots(2.14)$$

$$N(|G_x|, g) \le \beta_2\, exp\,[\lambda(t+x)] \quad \ldots(2.15)$$

$$N_0(|G|, g) \le \beta_3\, exp\,[\lambda(t+x)] \quad \ldots(2.16)$$

$$N_0(|G_x|, g) \le \beta_4\, exp\,[\lambda(t+x)] \quad \ldots(2.17)$$

$$N(|G_{xx}|, g) \leq \beta_5 \exp[\lambda(t+x)] \qquad ...(2.18)$$

where β_i, $i=1, \ldots, 5$ are nonnegative constants, λ is a positive constant and $(t, x) \in Q$.

(H_4) There exist nonnegative constants α_i, $i=1,\ldots,5$ such that

$$|h(x,t)| + M(|G|, |F(0)|) \leq \alpha_1 \exp[\lambda(t+x)] \qquad ...(2.19)$$

$$|h_x(x,t)| + M(|G_x|, |F(0)|) \leq \alpha_2 \exp[\lambda(t+x)] \qquad ...(2.20)$$

$$|h_t(x,t)| + M_o(|G|, |F(0)|) \leq \alpha_3 \exp[\lambda(t+x)] \qquad ...(2.21)$$

$$|h_{tx}(x,t)| + M_o(|G_x|, |F(0)|) \leq \alpha_4 \exp[\lambda(t+x)] \qquad ...(2.22)$$

$$|h_{xx}(x,t)| + M(|G_{xx}|, |F(0)|) \leq \alpha_5 \exp[\lambda(t+x)] \qquad ...(2.23)$$

where λ is a positive constant and $(x,t) \in Q$.

Our main results to be proved in this paper are embodied in the following theorems.

Theorem 1. Suppose that the hypotheses (H_1) - (H_4) hold and

$$\mu = \sum_{i=1}^{5} \beta_i < 1. \qquad ...(2.24)$$

Then there exists a unique classical solution $u \in S$ of the (MBVP) (1.1) - (1.3).

As an immediate consequence of Theorem 1 we have the following

Theorem 2. Assume that the hypotheses of theorem 1 hold. Then for any $u^{(0)} \in S$ the sequence $\{u^{(K)}\}$ given successively by

$$u^{(k)}(x,t) = h(x,t) + M(G, F(u^{(k-1)})) \qquad ...(2.25)$$

for $k=1, 2, \ldots$, converges in S to a unique classical solution u of the (MBVP) (1.1) - (1.3). Moreover

$$\left|u^{(k)} - u\right|_s \leq \frac{\mu^k}{1-\mu}\left|u^{(1)} - u^{(0)}\right|_s,$$

for $k = 1, 2, \ldots$, where μ is as defined in (2.24).

Proofs of theorems 1 and 2

For $u \in s$, we define the operator B by

$$Bu(x,t) = h(x,t)+M(G, F(u)), \tag{3.1}$$

then the equation (2.7) becomes the operator equation

$$u(x,t) = Bu(x, t). \tag{3.2}$$

Clearly, the solution of the operator equation (3.2) corresponds to the solution of the (MBVP) (1.1) - (1.3). Now we shall prove that B maps S into itself. From (3.1) and using (2.13), (2.19), (2.3), (2.5) and (2.14) we have

$$\begin{aligned}|Bu(x,t)| &\le |h(x,t)| + M(|G|,|F(0)|) + M(|G|,|F(u)-F(0)|)\\ &\le \alpha_1 \exp[\lambda(t+x)] + |u|_s N(|G|,g)\\ &\le [\alpha_1 + K\beta_1]\exp[\lambda(t+x)].\end{aligned} \tag{3.3}$$

Differentiating (3.1) with respect to the component x we have

$$(Bu(x,t))_x = h_x(x,t) + M(G_x, F(u)), \tag{3.4}$$

From (3.4) and using (2.13), (2.20), (2.3), (2.5) and (2.15) we obtain as in (3,3),

$$|(Bu(x,t)_x)| \le [\alpha_2 + K\beta_2]\exp[\lambda(t+x)]. \tag{3.5}$$

Differentiating (3.1) with respect to the component t we have

$$(Bu(x,t))_t = h_t(x,t) + M_0(G,F(u)). \tag{3.6}$$

From (3.6) and using (2.13), (2.21), (2.3), (2.5) and (2.16) we obtain as in (3.3),

$$|(Bu(x,t))_t| \le [\alpha_3 + K\beta_3]\exp[\lambda(t+x)]. \tag{3.7}$$

Differentiating (3.6) with respect to the component x we have

$$\left(Bu(x,t)\right)_{tx} = h_{tx}(x,t) + M_0\left(G_x, F(u)\right). \tag{3.8}$$

From (3.8) and using (2.13), (2.22), (2.3), (2.5) and (2.17) we obtain as in (3.3),

$$\left|\left(Bu(x,t)\right)_{tx}\right| \leq \left[\alpha_4 + K\beta_4\right]\exp\left[\lambda(t+x)\right]. \tag{3.9}$$

Differentiating (3.4) with respect to the component x we have

$$\left(Bu(x,t)\right)_{xx} = h_{xx}(x,t) + M\left(G_{xx}, F(u)\right). \tag{3.10}$$

From (3.10) and using (2.13), (2.23), (2.3), (2.5) and (2.18) we obtain as in (3.3)

$$\left|\left(Bu(x,t)\right)_{xx}\right| \leq \left[\alpha_5 + K\beta_5\right]\exp\left[\lambda(t+x)\right]. \tag{3.11}$$

Now from (2.1) and (3.3), (3.5), (3.7), (3.9), (3.11) we observe that

$$\left|\left(Bu(x,t)\right)\right|_E \leq \left[\sum_{i=1}^{5}\left[\alpha_i + K\beta_i\right]\right]\exp\left[\lambda(t+x)\right].$$

This shows that B maps S into itself.

Now we verify that the operator B is a contraction map. Let $u, \bar{u} \in S$. From (3.1) and using (2.13), (2.3), (2.14) we have

$$\left|Bu(x,t) - B\bar{u}(x,t)\right| \leq M\left(|G|, \left|F(u) - F(\bar{u})\right|\right) \leq \beta_1 \left|u - \bar{u}\right|_S \exp\left[\lambda(t+x)\right]. \tag{3.12}$$

Similarly, from (3.4), (3.6), (3.8), (3.10) and by making use of (2.3) and suitable conditions in hypothesis (H_3) we obtain

$$\left|\left(Bu(x,t)\right)_x - \left(B\bar{u}(x,t)\right)_x\right| \leq \beta_2 \left|u - \bar{u}\right|_S \exp\left[\lambda(t+x)\right], \tag{3.13}$$

$$\left|\left(Bu(x,t)\right)_t - \left(B\bar{u}(x,t)\right)_t\right| \leq \beta_3 \left|u - \bar{u}\right|_S \exp\left[\lambda(t+x)\right], \tag{3.14}$$

$$\left|\left(Bu(x,t)\right)_{tx} - \left(B\bar{u}(x,t)\right)_{tx}\right| \leq \beta_4 \left|u - \bar{u}\right|_S \exp\left[\lambda(t+x)\right], \tag{3.15}$$

$$\left|\left(Bu(x,t)\right)_{xx} - \left(B\bar{u}(x,t)\right)_{xx}\right| \le \beta_5 \left|u - \bar{u}\right|_S \exp\left[\lambda(t+x)\right]. \tag{3.16}$$

From (2.1) and using (3,12) - (3.16) we obtain

$$\left|\left(Bu(x,t)\right) - B\bar{u}(x,t)\right|_E \le \left(\sum_{i=1}^{5} \beta_i\right) \left|u - \bar{u}\right|_S \exp\left[\lambda(t+x)\right]. \tag{3.17}$$

Consequently, from (3.17) we have

$$|\ (Bu - B\bar{u}\ |_S \le \mu\ |\ u - \bar{u}\ |_S .$$

Since $\mu < i$, it follows from Banch fixed point theorem that B has a unique fixed point in S. The fixed point of B is however a classical solution of the (MBVP) (1.1) - (1.3). This completes the proof of Theorem 1.

Let $u^{(0)} \in S$ be given. Then we can determine a sequence $\{u^{(k)}\}$ successively from

$$u^{(k)}(x,t) = Bu^{(k-1)}(x,t), \qquad k = 1,2,\ldots \tag{3.18}$$

It is easy to observe from Theorem 1, the sequence determined from (3.18) converges to unique solution $u \in S$ of (3.2). Since (3.2) and (3.18) are the operator equations of (2.7) and (2.25) respectively, we conclude that the sequence $\{u^{(k)}\}$ given by (2.25) converges in S to the unique solution u of (2.7) and hence to the unique classical solution of (MBVP) (1.1) - (1.3). The error estimate (2.26) follows immediately from the contraction property of the operator B and the proof of Theorem 2 is complete.

FUTHER APPLICATIONS

In this section we indicate in brief the further applications of our approach to the study of different types of mixed boundary value problems for partial differential equations.

Recently, Ebihara [4], Pecher [14] and Lin [12] have studied respectively the following mixed type boundary value problems

$$\left.\begin{aligned} & u''-c\,\Delta u' = F(x,t,u,u_i,u_{ij},u',u'_i),\ x\in\Omega,\ t\le 0, \\ & u\,(x,0)=u_0(x),\ u'\,(x,0)=u_1(x),\ x\in\Omega, \\ & u(x,t)=\phi\,(x), x\in\Omega,\ t\ge 0; \end{aligned}\right] \quad ...(4.1)$$

$$\left.\begin{aligned} & \frac{\partial^2 u}{\partial t^2} - \Delta \frac{\partial u}{\partial t} + G(u,\nabla u,\nabla^2 u, \frac{\partial u}{\partial t}, \nabla\frac{\partial u}{\partial t})=0, \\ & u(x,0)=\phi\,(x),\ \frac{\partial u}{\partial t}(x,0)=\Psi\,(x) \end{aligned}\right] \quad .....(4.2)$$

and

$$\left.\begin{aligned} & u_{tt} = \left(\left(a_{ij}u_{x_i}\right)_{x_j}\right)_t + \left(b_{ij}u_{x_i}\right)_{x_j} + \left(F\left(x,t,u,u_x\right)\right)_t, \text{ in } Q_T, \\ & u(x,0)=u_0(x),\ u_t(x0)=u_1(x),\ x \text{ in } \Omega, \\ & u\,(x,t)=0 \text{ on } S_T \end{aligned}\right] \quad ...(4.3)$$

where all the function involved in (4.1), (4.2) and (4.3) are respectively as explained in [4], [14] and [12]. We note that our approach to the (MBVP) (1.1) - (1.3) can be applied equally well to study the existence, uniqueness and approximation of the solutions of the problems (4.1) - (4.2) by making suitable modifications in the difinitions of the spaces E and S in section 2 and by respresentiating the problems (4.1) - (4.3) into their equivalent integral equations by means of Green's functions and imposing the hypotheses similar to that of used in section 2 for the study of (MBVP) (1.1) - (1.3).

Finally we note that the method employed in this paper can also be used to deal with the following mixed type problems

$$u_{tt} - au_{xxt} = F_1(x,t,u,v,u_x,v_x,u_{xx},v_{xx}),\ x\in(0,L),\ t\in[0,T),$$

$$v_t - au_{xx} = F_2\,(x,t,u,v,u_x,v_x,u_{xx},v_{xx}),\ x\in(0,L),\ t\in[0,T],$$

$$u(x,0)=\phi\,(x),\ u_t(x,0)=\psi\,(x),\ v\,(x,0)=\xi\,(x),\ x\in(0,L).$$

$$u(0,t) = u(L,t) = 0,\ v(0,t) = v(L,t) = 0,\ t\in [0,T]; \qquad \dots(4.4)$$

and

$$u_{tt} - au_{xxt} = \int_0^t F(x,t,s,u(x,s),u_x(x,s),\ u_{sx}(x,s),\ u_{xx}(x,s))\, ds$$

$$+ f(x,t),\ x\in (0,L),\ t\in [0,L],$$

$$u(x,0) = \phi(x),\ u_t(x,0) = \psi(x),\ x\in [0,L],$$

$$u(0,t) = u(L,t) = 0,\ t\in (0,T] \qquad \dots(4.5)$$

where all the functions are defined on the respective domains of their definitions. For the study of slightly different versions of problems (4.4) and (4.5), see [11] and [5].

The precise formulations of all these results for the problems (4.1) - (4.5) similar to that of given in Theorems 1-2 and their proofs are very close to that of given in section 2 and 3 with suitable modifications, we do not discuss it here.

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AEROMYCOFLORA OF GURUKULA KANGRI PHARMACY, HARIDWAR

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ABSTRACT

Study of aeromycoflora of fermentation unit of Gurukula Kangri Pharmacy, Haridwar was carried out by gravity petridish method in the month of February and March, 1994. Percentage of abundance, frequency and variation in the density of total aeromycoflora was studied. The role of environmental factors i.e. relative humidity and temperature in affecting the densities of fungal propagules was taken into consideration. The fungal content of air at a particular sampling time varied between 7 and 66 propagules per 100 cm³. In the diurnal cycle, fungi showed an evening tendency in all samples except first sampling. The dominant species were *Cladosporium cladosporioides, Alternaria sp., Penicillium cyclopium, Epicoccum nigrum* and rest of the fungi were of sporadic occurrence.

Key Words : Aeromycoflora, diurnal cycle, *Cladosporium cladosporioides.*

INTRODUCTION

Aerobiological investigations into the microbial pollution of air of a ware house, poultry shed, sugar mill, hospital environment have a great influence on air spora of any place. Aerobiological taxonomy and density of its mycoflora exhibit variations with a change in weather conditions. The present investigation is related to aeromycoflora of fermentation unit of Gurukula Kangri Pharmacy at Haridwar.

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MATERIALS AND METHODS

Aeromycoflora of Gurukula Kangri Pharmacy was studied by using gravity petridish method during Feb-Mar, 1994. Four sets of petridishes (each having five plates) were exposed in Asav Department (Fermentation Unit) in the morning and evening separately for five minutes at two feet height from the ground level both for intramural (inside) as well as extramural (outside) studies. These plates were incubated at $25\pm1°C$ and the number of fungal colonies appearing were counted and identified. Relative humidity and temperature were recorded by placing hygrometer and thermometer at the sampling site.

RESULTS

The spore content of the air was rich both in quality and quantity. The percentage of abundance, variation in density of total aeromycoflora, percentage contribution of different fungi and percentage of frequency was given in (Table 1, 2, 3 and 4 respectively). A total of 12 fungal forms belonging to 8 genera were isolated.

The fungal content of the air at a particular sampling time varied between 7.0 and 66.0 propagules per 100 cm^3. Maximum number of propagules were trapped in first sampling at evening (outside) of room no.2.

In the diurnal cycle, fungi showed an evening tendency in the samples except in Ist sampling. Second peak was obtained at evening (inside). In all the other samplings morning gave minimum counts. The statistical analysis (Table-2b) showed insignificant variation in aeromycoflora. The dominant species showing a frequency above 75% were *Cladosporium cladosporioides and Alternaria* sp. Fungal species

with 50-75% frequency were *Penicillium cyclopium, Epicoccum nigrum* and white sterile form. Rest of fungi were of sporadic occurrence.

Cladosporium formed the main bulk of the total aeromycoflora, which was represented by its two species viz. *C. cladosporioides* and *C. herbarum. C. cladosporioides* was found to occur in the maximum number out of all the fungal propagules trapped by gravity petridish method in three samplings. Out of 842 colonies of *Cladosporium* trapped during the investigation, *C. cladosporioides* contributed 584 colonies.

Alternaria was the next abundant genus contributing 216 colonies to the total of 1705 colonies. White sterile form came next with a total of 188 colonies. *Penicillium* was represented by two of its species viz., *P. cyclopium* and *P. chrysogenum* which contributed 9.97% and 0.70% respectively to total aeromycoflora. *P. cyclopium* was the most frequent species. *Curvularia* sp. contributed 3.92% to the total aeromycoflora. *Epicoccum* was represented by one species i.e. *E. nigrum* which contributed 3.57% to the total aeromycoflora. The unidentified fungi were 3.34% in the aeromycoflora. The fungal species with less than 2% contribution included *Rhizopus* sp., *P. chrysogenum, Aspergillus niger* and *Trichothecium* sp. Besides one type of mycelia sterilia viz., white sterile form was also trapped from the aeromycoflora.

DISCUSSION

In the diurnal cycle highest quantities of fungi were trapped in the evening time and minimum during the morning except in case of Ist sampling when it was minimum in the evening. These findings are in conformity with those of Mishra and Kamal[2] and Sharma and Gupta.[5] The morning periods in winter months (February-March) are commonly

calm and the fungi most of which are wind disseminated are not disturbed during this period. The undisturbed enviornmental conditions accounted for a low fungal content of air in the morning, the fungal spores dry up and even gentle breeze which is common in the day time, disseminates the spores. The dissemination of spores results in the increased number of spores in the evening time and this was the reason why the peak was observed in the evening. The dominant fungal form in air were *Cladosporium, Alternaria, Aspergillus and Penicillia. Cladosporium* is an ubiquitous fungus and its dominance in fungal aerospora has been reported from different parts of the country.[1,6] During the course of present investigation, the total number of *Cladosporium* colonies were more in dry weather and showed a decline with the increase in humidity and decrease in temperature. *Alternaria* was isolated in abundance from air sampling of Gurukula Kangri Pharmacy. Paddy[4] also found this fungus very close in percentage to *Cladosporium.*

Aspergillus species were found to be present in the aeromycoflora throughout the period of investigation. Noble and Clayton[3] investigated the fungal flora of air of hospital ward and also found that *Aspergillus fumigatus* was the dominant type of spore.

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TABLE-1

ABUNDANCE (%) OF AEROMYCOFLORA OF GURUKULA KANGRI PHARMACY, HARIDWAR (FEB-MAR, 1994)

FUNGAL SPECIES	SAMPLING NUMBER AND TIME											
	I				II				III			
	A	B	C	D	A	B	C	D	A	B	C	D
Alternaria sp.	--	7.14	13.79	10.52	23.38	13.51	11.52	13.60	9.09	9.64	14.14	15.15
Aspergillus niger	4.76	1.78	--	--	5.64	1.35	--	--	4.54	--	3.88	3.03
Aspergillus sp.	19.04	--	3.44	--	0.80	--	--	0.59	--	0.87	--	--
Cladosporium cladosporioides	25.39	8.03	63.79	28.94	39.51	--	51.34	42.01	40.90	2.28	28.05	37.12
C. herbarum	1.58	--	--	--	--	28.37	12.60	14.79	--	9.47	33.05	--
Curvularia sp.	--	--	--	--	--	5.40	6.43	2.95	1.13	--	9.16	--
Epicoccum nigrum	--	--	8.62	--	3.22	12.16	5.09	6.50	3.40	0.87	1.94	1.51
Penicillium chrysogenum	--	--	1.72	--	--	--	2.41	--	--	--	0.55	6.81
P. cyclopium	44.44	47.32	6.89	39.47	6.45	9.45	--	1.77	15.90	16.66	11.94	9.09
Rhizopus sp.	4.76	--	--	--	--	--	--	--	--	--	--	--
Trichothecium sp.	--	0.89	--	2.63	--	2.74	3.75	2.36	--	2.63	0.27	0.75
White sterile form	--	25.89	1.72	2.63	17.74	27.02	--	--	25.00	17.54	6.66	26.51
Unidentified	--	8.92	--	15.78	3.22	--	3.21	14.79	--	--	--	--

A = Morning outside, B=Morning inside, C= Evening outside, D=Evening inside

TABLE-2A

DIURNAL VARIATION IN THE DENSITY OF TOTAL AEROMYCOFLORA PER 100 CM^3 OF GURUKULA KANGRI PHARMACY, HARIDWAR (FEB-MAR, 1994)

SAMPING TIME	SAMPLING NUMBER		
	I	II	III
Morning (Outside)	11.10	21.93	15.59
Morning (Inside)	19.80	13.09	20.16
Evening (Outside)	10.20	65.99	63.69
Evening (Inside)	6.72	29.90	23.35

TABLE-2B

TWO WAY ANALYSIS OF VARIANCE TABLE

Source of Variation	DF	Sum of Squares	Mean Square	F
Among means of treatment A	3	1049.19	349.73	1.58
Among means of treatment B	2	1870.81	935.40	4.25
Residual	6	1320.33	220.05	

A = Time Interval

B = Diurnal Cycle

DF = Degree of Freedom

Insignificant at 0.05 level

TABLE-3

PERCENTAGE CONTRIBUTION OF DIFFERENT FUNGI TO TOTAL AEROMYCOFLORA

FUNGAL SPECIES	FEB.-MAR., 1994
Alternaria sp.	12.66
Aspergillus niger	1.05
Aspergillus sp.	2.05
Cladosporium cladosporioides	34.25
C. herbarum	15.13
Curvularia sp.	3.92
Epicoccum nigrum	3.57
Penicillium chrysogenum	0.70
P. cyclopium	9.97
Rhizopus sp.	0.17
Trichothecium sp.	1.58
White sterile form	11.02
Unidentified	3.34

TABLE-4

FREQUENCY (%) OF DIFFERENT AEROMYCOFLORA OF GURUKULA KANGRI PHARMACY (FEB-MAR, 1994)

FUNGAL SPECIES	FREQUENCY (%)		
	I	II	III
Alternaria sp.	57.89	90.00	95.00
Aspergillus niger	26.31	20.00	45.00
Aspergillus sp.	31.57	10.00	05.00
Cladosporium cladosporioides	94.73	75.00	85.00
C. herbarum	05.26	55.00	45.00
Curvularia sp.	---	30.00	30.00
Epicoccum nigrum	15.78	75.00	50.00
Penicillium chrysogenum	05.26	20.00	25.00
P. cyclopium	84.21	55.00	70.00
Rhizopus sp.	15.78	---	---
Trichothecium sp.	10.52	40.00	20.00
White sterile form	36.84	45.00	95.00
Unidentified	26.31	40.00	---

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FLORISTIC STRUCTURE OF DAYARA BUGYAL : A HIGH ALTITUDE PASTURE IN DISTRICT UTTARKASHI OF GARHWAL HIMALAYA

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ABSTRACT

Dayara Bugyal, a high altitude pasture in Bhatwari Block of District Uttarkashi, is situated just above the timberline at an altitude of 3100-3500 m and occupies 5x10 km area. Dayara Bugyal is very rich in its biodiversity. The exploration of this pasture yielded 269 plant species belonging to 52 families and 162 genera. Out of the total species recorded, 216 were Dicotyledons, 43 Monocotyledons, 6 Pteridophytes and 4 belonged to Gymnosperms. The dominant family was Asteraceae whereas, the dominant genera were *Potentilla, Swertia* and *Polygonum.*

Key words : Bugyal, high altitude pasture, Garhwal Himalaya, flowering plants, floristic composition, biotic interference.

INTRODUCTION

'Bugyals', the high altitude alpine pastures compose an important part of landscape of the Garhwal Himalaya (77° 33.5' E to 80° 6.0' E longitude and 29° 31.9' N to 31° 26.5' N latitude). Amongst important bugyals of the Garhwal Himalaya, Tapovan, Nandanvan, Har-ki-Doon, Ginda, Gidara and Dayara are situated in District Uttarkashi (30° 22' N latitude and 75° 51' E longitude). These alpine maedows have been a continuing source of attraction to human beings for their beautiful and fragrant flowering plants. They have been subjected to intense grazing during summers due to high productivity of forage herbs for centuries.

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The studies on high altitude pastures of the Himalaya have been made by various workers [1-14], but detailed floristic exploration of Dayara Bugyal of Uttarkashi district has not been done.

THE STUDY AREA

Location and Physiography :

Dayara Bugyal occupies 5x10 km area in Bhatwari Block of Distruct Uttarkashi, at an altitude ranging from 3100 to 3500 m. It is situated just above the treeline and remains snowclad from December to late March. Bhatwari, a town 28 km far from District Headquaters, Uttarkashi, is the place from where the trekking route for Dayara Bugyal starts. Dayara Bugyal is 15 km far from Bhatwari which can be reached either via village Raithal or via villages Pala and Barsu.

The steep slope starting from Raithal becomes gentler at the base of Dayara Bugyal with elevations and depressions providing ideal condtions for winter games. At certain places water oozes out forming streamlets which after uniting form a beautiful perennial stream. Dayara Bugyal is devided into two prominent areas by a ridge, occupied by few tree species. These areas are Dayara proper and area leading to the Bakaria Top. The whole area remains occupied by herbs and grasses except few patches of trees in Dayara proper and on ridge, and by shrubs in basal portion of Dayara Bugyal.

Climate :

Being situated in alpine zone, Dayara Bugyal exhibits a colder climate. The whole area remains covered with snow from November to mid March. From March to October, morning remains pleasent with bright sunshine but in the afternoon the area usually becomes foggy and gets rains. The temperature of the study area suddenly falls with rains. The temperature varies from 2°-8° C during September-October; 7°-20°C

during May-June and falls even below 0° during November to February. The annual rainfall varies from 3000 mm to 5000 mm and the relative humidity is too high.

Soil :

Soil of the area is clayey loam which is very rich in organic matter. Nitrogen content of the soil is very high. The rocks and boulders play an important role in supporting the plant species which help in the soil formation.

Vegetation:

The route leading to Dayara Bugyal is dominated by montane vegetation which is composed of *Quercus leucotrichophora, Q. semecarpifolia, Q. dilatata, Rhododendron campanulatum, R. arboreum,* just above of Raithal; *Abies pindrow, Picea morinda, Cupressus torulosa, Cedrus deodara, Taxus baccata, Betula utilis* above half of the way and *Acer caesium, A.pictum, Buxus wallichiana, Aesculus indica, Juglans regia, Ulmus wallichiana* etc. Just below of Dayara Bugyal.

Dayara Bugyal is occupied mostly by herbaceous plant species except some trees like *Picea morinda* and few dwarf scrubs like *Berberis kumaonensis, Crotoneaster microphylla, Juniperus communis, Lonicera obovata, Rhododendron anthopogon* and *R. lepidotum.* Some of the well known plants of Dayara Bugyal are species of *Anemone, Corydalis, Gentiana, Geranium, Iris, Meconopsis, Morina, Pedicularis, Polygonum, Ranunculus, Aconitum, Saussurea, Saxifraga* and *Sedum* which give beautiful mosaic patterns.

Biotic Interference :

Due to high productivity in relation to forage herbs, the area is visited by 'Gujjars' of Tarai region, shepherds and nearby villagers with all their buffaloes, sheep, goats, horses and other cattle. Their visit even

in November may be evidenced by the presence of mandaps of 'Gujjars' and 'Chhanis' of nearby villagers. Thousands of cattle are brought to the area in April-May and carried back in September when temperature begins to fall. The vegetation of the area gets adversely affected by grazing, overgrazing, trampling, uprooting, excreta shedding etc.

Floristic Composition :

Asteraceae with 34 plant species was the dominant family of Dayara Bugyal. This was followed by Rosaceae, Ranunculaceae, Polygonaceae, Poaceae, Gentianaceae, Lamiaceae, Liliaceae and Fabaceae consisting of 19, 16, 15, 14, 13, 12, 10 and 09 species respectively. However, *Potentilla, Swertia and polygonum* each with 08 species were dominant genera of the area. The statistical analysis of the taxa collected (Table : 1) shows monocot-dicot ratio of 1:5.

Table -1. Statistical analysis of taxa collected from Dayara Bugyal

Group	Families	Genera	Species
Dicotyledons	39	128	216
Monocotyledons	08	28	43
Gymnosperms	02	03	04
Pteridophytes	03	03	06
Total	52	162	269

SYSTEMATIC ENUMERATION

This work is a record of collection of plant species, made from Dayara Bugyal, during the visit in early October 1988 and early September 1989 and then at regular intervals during April 1992 to November 1992 and during

late March 1993 to late October 1993. Each plant specimen collected was identified [15-17] and enumerated. The speciments of collected plant species have been preserved in the Herbarium of Botany Department, Government Post Graduate College, Uttarakashi.

Ranunculaceae

Aconitum heterophyllum Wallich ex Royle
A. violaceum Jacq. ex Stapf.
Adonis chrysocyathus Hook. f. & Thoms.
Anemone rivularis Buch.-Ham. ex DC.
A.vitifolia Buch.-Ham. ex DC.
Aquilegia nivalis Falc. ex Jackson
Caltha palustris Linn.
Delphinium caeruleum Jacq. ex Camb.
D. cashmerianum Royle
D. vestitum Wallich ex Royle
Ranunculus arvensis Linn.
R. hirtellus Royle ex D.Don.
R. sceleratus Linn.
Thalictrum alpinum Linn.
T. foliolosum DC.
T. virgatum Hook. f. & Thomas

Berberidaceae

Berberis asiatica Roxb. ex DC.
B. kumaonensis Schneld.
B. Lycium Royle

Podophyllaceae

Podophyllum hexandrum Royle

Papavaraceae

Meconopsis aculeata Royle
M. paniculata Prain
Papavar dubium Linn.

Fumariaceae

Corydalis cornuta Royle

C. cashmeriana Royle

C. govaniana Wallich

Dicentra macrocapnos Prain

Fumaria indica (Hassk.) Pugsley

Brassicaceae

Arabidopsis himaliana (Edgew.) Schulz.
Capsella bursa-pastoris (L.) Medik.
Cardamine macrophylla willd.

Violaceae

Viola biflora Linn.

Caryophyllaceae

Areneria festucoides Benth.
Gypsophylla cerastioides D.Don.
Silene laxantha Majumdar
S. viscosa (L.) Pers.
Stellaria decumbens Edgew.

Hypericaceae

Hypericum monanthemum Hook. f. Thoms.
H. oblongifolium Choisy

Malvaceae

Abelmoschus manihot (L.) Medik

Geraniaceae

Geranium nepalense Sweet
G. wallichianum D. Don ex Sweet

Balsaminaceae

Impatiens bicornuta Wallich
I. gigantea Edgew.
I. glandulifera Royle
I. sulcata Wallich
I. thomsonii Hook. f.

Rutaceae

Boenninghausenia albiflora (Hook). Reichb. ex Meisn.
Skimmia laureola (DC.) Sieb. & Zucc. ex Walp.

Aquifoliaceae

Ilex dypyrena Wall.

Fabaceae

Astragalus leucocephalus Garh. Ex Benth.
A. melamostachys Benth. ex Bunge
Crotalaria albida Heyne ex Roth.
C. sessiliflora Linn.
Desmodium confertum DC.Prodr.
D. microphyllum (Thunb.) DC.
Indigofera heterantha Wall. ex Brandis
Lespedeza gerardiana Grah. ex Maxim.
Smithia ciliata Royle

Rosaceae

Crotoneaster acuminata Lindl.
Fragaria nubicola Lindl.
F. vesca Linn.
Geum elatum Hook. f.
Potentilla astrosanguinea Lodd.
P. cuneata Wallich ex Lehm.
P. fruticosa Linn. Var. pumila Hoo. f.
P. fulgans Wall. ex Hook.
P. microphylla D. Don
P. nepalensis Hook.
P. peduncularis D.Don.
P. polyphylla Wallich ex Lehm.
Pyracantha crenulata (D. Don.) Roem.
Rosa macrophylla Lindl.
R. sericea Lindl.
Rubus ellipticus Smith
R. nepalensis (Hook. f.) kuntze.
Sorbus foliolosa (Wall.) Spach.
Spiraea bella Sims.

Saxifragaceae

Bergenia ciliata (Haw.) Sternb.
B. stracheyi (Hook. f. & Thoms.) Endl.
Saxifraga andersonii Engl.
S. diversifolia Wall. ex Sering
S. jacquemontiana Dcne.
S. stenophylla Royle

Parnassiaceae

Parnassia cabulica Planchon ex Clarke
P. nubicola Wallich ex Royle.

Crassulaceae

Rhodiola heterodonta (Hook. f. & Thoms.) Boriss.
Sedum multicaule Wallich ex Lindl.

Melastomataceae

Osbeckia stellata Buch.-Ham. ex D. Don.

Onagraceae

Epilobium latifolium Linn.
E. laxum Royle
E. parvifolium Schreb.
E. royleanum Hausskn.

Apiaceae

Angelica glauca Edgew.
Bupleurum candollei Wallich ex DC.
B. gracillimum Klotzscb
Pimpinella diversifolia DC
Selinum tenuifolium Wallich ex C.B. Clarke
S. vaginatum Clarke
S. wallichianum (DC.) Raizada & Saxena
Trachydium roylei Lindl.

Caprifoliaceae

Lonicera myrtillus Hook. f. & Thoms.
Viburnum cotinifolium D. Don.

Rubiaceae

Galium hirtiflorum Req.
G. trifolium Requien ex DC.
Oldenlandia coccinia Royle
Rubia cordifolia Linn.

Valerianaceae

Valeriana himalayana Grub.
V. jatamansii Jones.

Dipsacaceae

Dipsacus inermis Wall.
Morina longifolia Wallich ex DC.

Asteraceae

Anaphalis adnata DC.
A. busua (Buch.-Ham. ex D.Don.) DC.
A. contorta (D.Don) Hook. f.
Aster diplostephioides (DC.) C.B. Clarke
A. falconeri (C.B. Clarke) Hutch.
A. peduncularis Wall. ex Nees
Artemisia gmelinii Web. ex Stechm.
A. stricta Edgew.
Bidens biternata (Lour.) Merr. et Sherff
Erigeron alpinus Linn.
E. multiradiatus (Lindl. ex DC.) Clarke
Gnaphalium affine D. Don.
Inula cappa (Buch.-Ham. ex D. Don) Clarke
I. royleana DC. Prodr.
Leontopodium alpinum Cass.
Ligularia amplexicaulis DC.

Myriactis wallichii Less
Onopordum acanthium Linn.
Picris hieraciodes Linn.
Saussurea costus (Falc.) Lipsch.
S. fastuosa (Decne) Sch. Bip.
S. gossipiphora D. Don.
S. graminifolia Wall. ex Dc.
S. roylei (DC.) Sch. Bip.
Senecio cappa Buch.-Ham. ex D.Don.
S. Chrysanthemoides DC.
S. graciflorus DC.
S. wallichii DC.
Sonchus oleraceous Linn.
Tanacetum dolychophyllum (Kitam.) Kitam.
T. nubigenum Wallich ex DC.
Taraxacum officinale Wiggers.
Youngia japonica (Linn.) DC.

Campanulaceae

Campanula argyrotricha Wall. ex DC.
Cyananthus lobatus Wall. ex Benth.
Lobelia pyramidalis Wall.

Ericaceae

Gaultheria trichophylla Royle
Rhododendron anthopogon D.Don.
R. arboreum Smith
R. campanulatum D. Don.
R. lepidotum Wall. ex G. Don.

Primulaceae

Androsace lanuginosa Wallich
A. Primuloides Duby
Primula denticulata Sm.
P. stuartii Wallich

Gentianaceae

Gentiana capitata Buch.-Ham. ex D. Don.
G. pedicellata (D.Don.) Griseb.
G. stipitata Edgew.
G. venusta (G. Don.) Griseb.
Halenia elliptica D. Don.
Swertia alata Royle ex Clarke
S. alternerifolia (Royle ex D. Don) Clarke
S. chirata Buch.-Ham.
S. cuneata D. Don.
S. hookeri C.B. Clarke
S. nervosa (G.Don.) C.B. Clarke
S. petiolata D. Don.
S. speciosa D. Don.

Boraginaceae

Cynoglossum glochidatum Wallich ex Benth.
Eritrichium minimum (Brand) Hara
Lindelofia stylosa (Karelin & Kir) Brand.
Myosotis caespitosa Schultz

Scrophulariaceae

Hemiphragma heterophyllum Wall.
Lindenbergia grandiflora (Buch.-Ham. ex D. Don.) Benth
Pedicularis bicornuta Klotzsch

P. pectinata Wallich ex Benth.
Picrorhiza kurrooa Royle ex Benth
Scrophularia calycina Benth
Verbascum thapsus Linn
Veronica cana Wall. ex Benth.

Acanthaceae

Pteracanthus alatus (Wall. ex Nees) Bremek.

Lamiaceae

Ajuga parviflora Benth.
Elsholtzia fruticosa (D. Don.) Rehder.
Lamium album linn.
Leucus lanata Benth.
Mentha longifolia (L.)Hudson
Micromeria biflora (Buch.-Ham. ex D.Don.) Benth.
Nepeta govaniana (Benth.) Benth.
Phlomis macrophylla Wallich ex Benth.
P. rotata Benth. ex Hook. f.
Prunella vulgaris Linn.
Salvia nubicola Wall. ex Smith
Scutellaria angulosa Benth.

Polygonaceae

Bistorta affinis (D.Don.) Green
B. macrophylla (D.Don.) Sojak
B. Vaccinifolia (Wallich ex Meiss.) Green
Oxyria digyna (Linn.) Hill.
Periscaria polystachya (Wall. ex Meiss.) Gross
Polygonum alatum Buch.-Ham.
P. alpinum All.

P. amplexicaule D.Don.
P. barbatum Linn.
P. delicatulum Meissn.
P. nepalense Meissn.
P. sphaerocephalum Wall. ex Meissn.
P. vaccinifolium Wall. ex Meissn.
Rumex acetosa Linn.
R. nepalensis Sprengel.

Euphorbiaceae

Euphorbia cognata (Klotz. & Clarke) Boiss.
E. stracheyi Boiss.

Urticaceae

Elatostema pusillum Clarke ex Hook. f.
Gerardinia diversifolia (Link) Friis.
Pilea umbrosa Blume

Betulaceae

Betula utilis D. Don.

Orchidaceae

Dactylorhiza hatagirea (D.Don.) Soo
Diphylax griffithii (Hook. f.) Krzl.
Eulophia dabia (D.Don.) Hochr.
Goodyera fusca (Lindley) Hook. f.
Satyrium nepalense D.Don.

Zingiberaceae

Roscoea alpina Royle
R. purpurea Smith

Iridaceae

Iris kemaonensis D.Don. ex Royle

Liliaceae

Allium ceasium Schrenk.
A. wallichii Kunth.
Asparagus filicinus Buch.-Ham. ex D.Don.
Lilium nepalense Kunth.
L. oxypetalum (D. Don) Baker
Paris polyphylla Smith
Polygonatum multiflorum (Linn.) All.
P. verticillatum (Linn.) All.
Smilacina purpurea Wallich
Trillidium govanianum (D.Don) Kunth.

Juncaceae

Juncus himalensis Klotzsch
J. leucanthus Royle ex D.Don.

Araceae

Arisaema propinquum Schott.
A. tortuosum (Wall) Schott.

Cyperaceae

Carex alpina Sw.
C. filicina Nees
C. nubigena D. Don
C. setigera D. Don.
Fimbristylis pierotti Miq.
Kobresia duthiei Clarke
K. nepalensis (Nees) Kukenth.

Poaceae

Agrostis munroana Ait. et Hemsl.
A. pilosula Trin.
Avena subspicata Clairv.
Danthonia cachemyriana Jauh. Spach.
D. Jacquemontii Bor
Festuca gigantea (Linn.) Vill.
Pennisetum flaccidum Griseb.
P. glaucum (Linn.) R.B.
Phalaris minor Retz.
Pheleum alpinum Linn.
Poa annua Linn.
P. pratensis Linn.
P. Supina Schrad.
Trisetum scitulum Bor.

Pinaceae

Abies pindrow Royle
Picea smithiana (Wall.) Boiss.

Cupressaceae

Juniperus communis Linn.
J. macropoda Boiss.

Adiantaceae

Adiantum pedatum Linn.
A. venustum Linn.

Aspleniaceae

Asplenium dalhousiae Hook.
A. ensiforme Wall.

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A MODEL OF MAGNETORADIATIVE STRONG SHOCK WAVE IN UNIFORM ATMOSPHERE

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ABSTRACT

Self-similar solution has been investigated behind the propagating magnetoradiative spherical shock wave in uniform atmosphere. An idealized azimuthal magnetic field and radiative heat flux have been taken into consideration but radiation pressure and energy have been ignored. Total energy of wave remains constant.

Key-Words : Uniform atmosphere, Shock wave, Radiation, Magnetic field.

INTRODUCTION

Elliot [1] has studied the explosion problem including radiation heat flux in uniform atmosphere. Helliwell [2] has considerd the piston problem with radiation heat flux with varying density. Singh [3] has discussed strong magnetogasdynamic spherical shock wave in uniform atmosphere with increasing energy. Summers [4] and Rosenau and Frankenthal [5] have obtained the similarity solution of the flow variables behind propagating spherical shock wave in the presence of an idealized magnetic field in non-uniform atmosphere with constant and increasing energy respectively.

Recently Singh and Vishwakarma [6] and Singh and singh [7] have investigated their problems with radiation flux, magnetic field and gravitational force using similarity method. Gretler and Steiner [8] and Gretler [9] have also discussed blast waves in inhomogeneous media taking into consideration the effects of counter pressure and heat transfer in detail without considering the effects of magnetic field. Here total energy behind the strong shock surface is assumed to be constant. Summers model of strong spherical shock wave has been taken into

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account in which he supposed an idealized magnetic field such that lines of forces lie on hemisphere whose centre is the point of explosion. The equations are considered for only a hemisphere enclosing the origin.

The strong shock is assumed to advance into a conducting gas of constant density and pervaded by a spatially decreasing magnetic field tangential to the advancing shock front. All the assumptions regarding the radiation phenomena have been borrowed from Elliot's problem (1) of radiation gas dynamics.

The singularity at the centre of the disturbance was automatically removed with the introduction of radiation flux, but by introducing magnetic field it exists. Numerical calculations have been made using integration method to calculate the value of β (density ratio at the shock front) by Runge-Kutta method.

The figures given in the last section disclose the nature of flow variables for different values of non-dimensional parameters involved in the problem Viscosity and Gravitational forces have been ignored.

EQUATION OF MOTIONS AND BOUNDARY CONDITIONS

The equations of continuity, motion, magnetic field and energy for spherical symmetry following Summers [4] and Elliot [1] are as follows.

$$\frac{d\rho}{dt}+\frac{\rho}{r^2}+\frac{\partial}{\partial r}\left(ur^2\right) = 0, \tag{1}$$

$$\rho\frac{du}{dt} = -\frac{\partial\rho}{\partial r}-\frac{\mu H}{r}\frac{\partial}{\partial r}(rH), \tag{2}$$

$$\frac{dH}{dt}+\frac{H}{r}\frac{\partial}{\partial r}(ru) = 0, \tag{3}$$

$$\frac{dE}{dt}+\rho\frac{d}{dt}\left(\frac{1}{\rho}\right)+\frac{1}{\rho r^2}\frac{\partial}{\partial r}\left(Fr^2\right) = 0, \tag{4}$$

$$\text{where } F = -\frac{c\lambda}{3}\frac{\partial}{\partial r}\left(aT^4\right), \tag{5}$$

$$E = -\frac{p}{\rho(\gamma - 1)}, \tag{6}$$

$$p = \Gamma \rho T \tag{7}$$

$$\text{and } \frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r},$$

ac/4 is the Stefan-Boltzmann constant, c is the velocity of light and λ is the temperature, Γ the gas constant and γ is the ratio of specific heat. E, being the internal energy per unit mass, p the material pressure, u the velocity, ρ the density and F the heat flux in local thermodynamical equilibrium and H is transverse magnetic field.

In this problem, the disturbance is bounded by strong shock front. The four boundary conditions at the shock surfaces are given by principle of conservation of mass, momentum, magnetic field and energy are as

$$\rho_1(V - u_1) = \rho_2(V - u_2) = m_s(say), \tag{8}$$

$$\left(p_1 + \frac{\mu H_1^2}{2}\right) - \left(p_2 + \frac{\mu H_2^2}{2}\right) = m_s(u_1 - u_2) \tag{9}$$

$$H_1(V - u_1) = H_2(V - u_2) \tag{10}$$

$$\text{and } E_1 + \frac{p_1}{\rho_1} + \frac{1}{2}(V - u_2)^2 + \frac{\mu H_1^2}{\rho_1} - \frac{F_1}{m_s}$$

$$= E_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(V - u_2)^2 + \frac{\mu H_2^2}{\rho_2} - \frac{F_2}{m_s} \tag{11}$$

where suffixes 1 and 2 denote conditions immediately behind and ahead of the shock front respectively and V is the shock velocity.

For strong shock into the ambient material

$\rho_2 = \rho_a$, $H_2 = H_a$, $F_2 = p_2 = u_2 = 0$; then

equations (8)-(11) reduce to

$$u_1 = \left(\frac{\beta - 1}{\beta}\right) V, \tag{12}$$

$$p_1 = \left[\frac{\beta - 1}{\beta} + \frac{1 - \beta^2}{2M_A^2}\right] \rho_a V^2, \tag{13}$$

where $\beta = \dfrac{\rho_1}{\rho_a} = \dfrac{H_1}{H_a}$, (14)

if $E = \dfrac{p}{\rho(\gamma - 1)}$, then

$$F_1 = (\beta - 1)\left[\frac{\{(\gamma+1)/(\gamma-1)\} - \beta}{2\beta^2} - \frac{1}{M_A^2}\right] \rho_a V^2, \tag{15}$$

M_A denotes Alfven mach number and is given by

$$M_A = V / V_A = V / \left(\frac{\mu H^2}{\rho}\right)^{1/2}$$

where V_A is Alfven speed.

SIMILARITY SOLUTIONS

Equations (1) to (4) can be reduced to four ordinary differential equations provided that the variables are of similarity form

$$u = Ru(x), \qquad \rho = \rho_a \bar{\rho}(x),$$

$$p = \rho_a R^2 \overline{p}(x), \qquad E = R^2 \overline{E}(x), \tag{16}$$

$$F = \rho_a R^3 \overline{F}(x), \qquad \mu^{1/2} H = \rho_a{}^{1/2} R \overline{H}(x),$$

where ρ_a & H_a are reference density and magnetic field respectively given by

$$\rho_a = \text{constant} \tag{17}$$

and $H_a = H_e R^{-3/2}$ (18)

(following Summers [4])

H_e is constant; $x=r/R$ and R is the shock radius which is a function of time only given by

$$R^2 = A^2 R^{-\alpha} \tag{19}$$

a and A being constant to be determined by rate of changes of total energy of the disturbance. Assume the disturbance lies between hR and R where h is a constant and $0 \leq h < 1$, then the total energy will be given as

$$E_r = 4\pi \int_{hR}^{R} \left(E + \frac{1}{2}u^2 + \frac{\mu H^2}{2\rho} \right) \varsigma r^2 dx, \tag{20}$$

which after using similarity transformations (16) becomes

$$E_r = 4\pi \rho_a R^2 r^3 \int_h^1 \left(\overline{E} + \frac{1}{2}\overline{u}^2 + \frac{\mu \overline{H}^2}{2\overline{\rho}} \right) x^2 dx,$$

i.e. $E_T = \rho_a B R^2 R^3$, (21)

where B is constant.

If the total energy E_T of the system is conserved then by comparing

(19) with (21) we get,

$$\alpha = 3, \tag{22}$$

and $A^2 = E_T / \rho_a B = R^2 R^3$ (23).

By using similarity transformation into equation (1) to (5) we have.

$$\left(\bar{u} - x\right)\left(\overline{\rho'} / \bar{\rho}\right) = -\left[\left(2\bar{u} / x\right) + \bar{u}\right], \tag{24}$$

$$\overline{\rho'} / \bar{\rho} = (3/2)\bar{u} - \left[\left(\bar{u} - x\right) - \overline{H}^2 / \bar{\rho}\left(\bar{u} - x\right)\overline{u'} - \overline{H}^2 / 2\bar{\rho}\left(\bar{u} - x\right)\right] \tag{25}$$

$$\left(\bar{u} - x\right)\left(\overline{H'} / \overline{H}\right) = 3/2 - \overline{u'} - \bar{u} / x \tag{26}$$

$$\left(\bar{u} - x\right)\overline{E'} - 3\overline{E} + \left(\bar{p} / \bar{\rho}\right)\left[\left(2\bar{u} / x\right) + \overline{u'}\right] + \frac{1}{\rho x^2}\frac{d}{dx}\left(\overline{F}x^2\right) = 0, \tag{27}$$

where prime (') denotes differentiation with respect to x.

In place of the energy equation (27) it is possible to obtain a first integral.

$$\overline{F} = \left(x - \bar{u}\right)\bar{\rho}\left[\left\{\bar{p} / \bar{\rho}(\gamma - 1)\right\} + \frac{1}{2}\overline{u^2}\right] - \overline{pu} \tag{28}$$

For expression (5) for radiation diffusion to be of similarity from the mean free path of radiation must vary as

$$\lambda = \lambda_1 T^{-(17/6)} g\left(\rho / \rho_a\right), \text{ following Elliot [1],} \tag{29}$$

From mean free path data we take

$$g\left(\rho / \rho_a\right) = \rho_a / \rho = 1 / \bar{\rho}$$

Then by using (6), (7) and (5) we get

$$\overline{F} = -K\overline{p}^{(1/6)}\overline{\rho}(-19/6)\left[\overline{p'}\overline{\rho} - \overline{\rho'}\overline{p}\right] \quad (30)$$

where $K = (4ac\lambda_1 / 3\rho_a)\Gamma^{-7/6} A^{-2/3}$

Equation (24), (25), (26) and (30) yield

$$\overline{u'}\left[(\overline{u}-x) - \overline{H}^2 / \overline{\rho}(\overline{u}-x) - \overline{p} / \rho(\overline{u}-x)\right]$$

$$= \overline{F}\overline{p}^{-7/6} / K\overline{\rho}^{(1/6)} + 3\overline{u}/2 - \overline{H}^2 / 2\overline{\rho}(\overline{u}-x) + 2\overline{u}\overline{\rho} / x\overline{\rho}(\overline{u}-x) \quad (31)$$

where $\overline{F}$ is given in (28).

If we put terms in their similarity form, (12) to (15) reduce to

$$\overline{u}_1 = (\beta - 1)/\beta , \quad (32)$$

$$\overline{p}_1 = (\beta - 1)/\beta + (1-\beta^2)/2M_A^2 \quad (33)$$

$$\overline{\rho}_1 = \beta , \quad (34)$$

$$\overline{H}_1 = \beta / M_A \quad (35)$$

and $$\overline{F}_1 = (\beta - 1)\left[\frac{\{(\gamma+1)/(\gamma-1)\} - \beta}{2\beta^2} - 1/M_1^2\right] \quad (36)$$

NUMERICAL CALCULATIONS AND RESULTS

Numerical ingegration using Runge-Kutta method has been obtained.

For Strong condition in equations (33) and (36) for $\overline{p}_1$ and $\overline{F}_1$, the second terms may be ignored because their values are very small in comparison to 1.

Hence,

$$\bar{p}_1 = (\beta - 1)/\beta$$

$$\text{and } \bar{F}_1 = (\beta - 1)\left[\frac{\frac{\gamma+1}{\gamma-1} - \beta}{2\beta^2}\right] \tag{37}$$

$$\text{Therefore, } \bar{u}_1 = \bar{p}_1 = \frac{\bar{\rho}_1 - 1}{\bar{\rho}_1} \tag{38}$$

The equations are ingegrated by using shock relations (35), (37) and (38).

We have calculated our problem for the different values of the non-dimensional constant parameters involved in the present problem. They are

$\gamma = 1.2, \; M_A = 10{,}100;$

$\bar{K} = 0.5, \; 10, \; 100.$

Here we have

$$E_1 = B\rho_a \dot{R}^2 R^3 \tag{39}$$

$$\text{where } B = 4\pi\int_0^1 \left[\frac{\bar{p}}{(\gamma-1)\rho} + \frac{1}{2}\bar{u}^2 + \frac{\bar{H}^2}{2\rho}\right]\bar{\rho}x^2 dx \tag{40}$$

We can calculate our β for initial conditions for the range $0 < 1/\beta < 1$ by integration using the equation of conservation of mass in Lagrangian form. Integrating this equation between the limits R and r^* $(0<r^*>R)$ and writing the resultant expression in non dimensional form we get.

$$\frac{1}{3} = \int_x^1 x^2 \, \bar{\rho} dx \tag{41}$$

where $x^* = r^* / R$ and r^* is Eulerian coordinate of contact discontinuity surface at which kinematics condition

$$\bar{u}(x^*) = x, \tag{42}$$

is satisfied. The kinematics condition discloses that the velocity of the fluid particle at the expanding surface is equal to the velocity of the surface itself.

There is a formation of cavity around the centre in the presence of magnetic field in our problem. Fig. 1 shows that velocity is of increasing order in the presence of the field whereas it decreases in the absence of the field and range of variation is $x = 1$ to $x = 0$.

In absence as well as in presence of magnetic field pressure always decreases. The range of variation of pressure in absence of field is greater than its variation with magnetic field.

In every case, with magnetic field, the range of variation is $x = 1 \, to \, x = 0.7$ but, in absence of magnetic field the range is from 1 to 0. The nature of variation of pressure is given in Fig. 2.

In density distribution case, shown in Fig. 3, the variation is of decreasing nature in both of the cases. In this case also the range of variation is from $x = 1 \, to \, x = 0.7$ in presence of magnetic field while, without magnetic field, range is from $x = 1$ to $x = 0$.

Fig. 4, reveals that the magnetic field distribution increase very rapidly between $x = 1 \, to \, x = 0.7$. So for as radiation flux distribution is concerned there is sharp difference between both the cases (i.e. with and

without magnetic field). In fig. 5, we have plotted flux-variation only for $K = 100$ for both the cases.

When we consider $K = 0.5$ and Alfven' Mach Number $M_A = 10$, flux increases slowly for a very thin range but when $k = 0.5$ and $M_A = 100$, flux variation decreases slowly upto to $x = 0.99$ to $x = 0.95$ it becomes negative which reveals that between this range there is absorption of radiation. While we choose $K = 10$, $M_A = 10$ & 100 it decreases slowly in a very small range. Thus the range of variation is the same $(\text{i.e. } x = 1 \text{ to } x = 0.7)$ in presence of field. When $K = 100$, the ratio of the flux decreases very fast upto $x = 0.910$ towards the centre after that it decreases on the same pattern upto $x = 100$ for $K = 10$. In absence of the field the flux ratio increases very very fast for $K = 0.5$ upto $x = 0.699$ and than it decreases slowly upto $x = 0$. The conclusion is that there is a much difference between the values of flux with and without magnetic field.

In the above discussions the results of Elliot [1], Greatler and Steiner [8] and Gretler [9] which were obtained in ordinary gases without magnetic field have been compared with our present results which are obtained in the presence of magnetic field. In the discussions the words "with and without magnetic field" show the present results and the results obtained earlier respectively.

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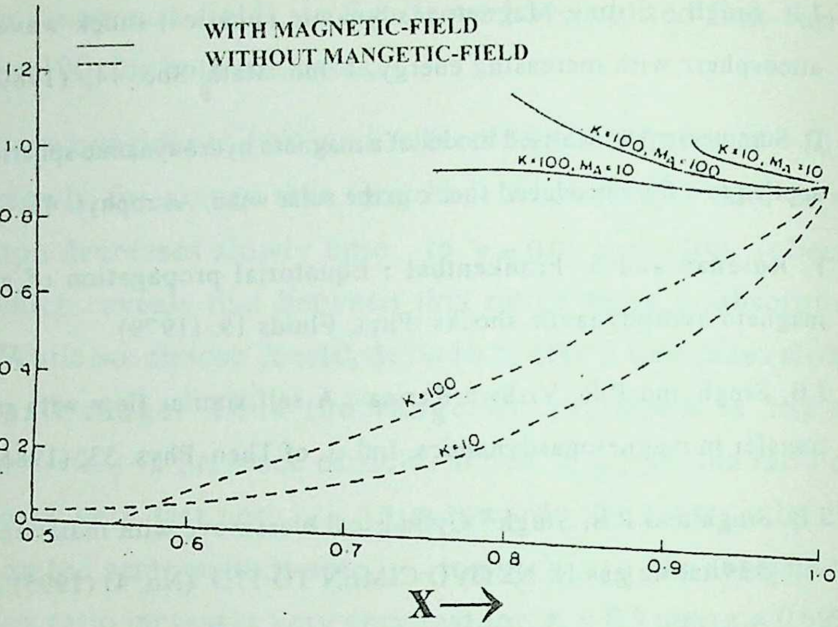

Fig. 1 Velocity distribution

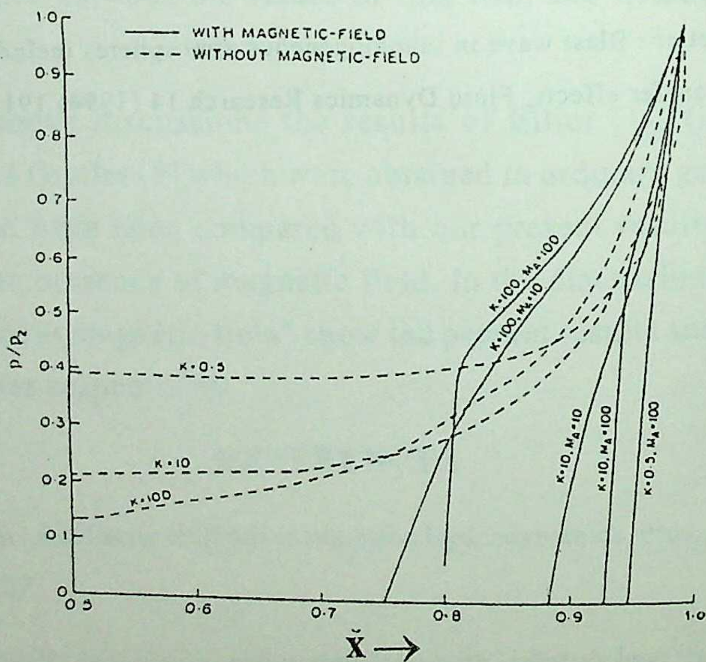

Fig. 2 Pressure distribution

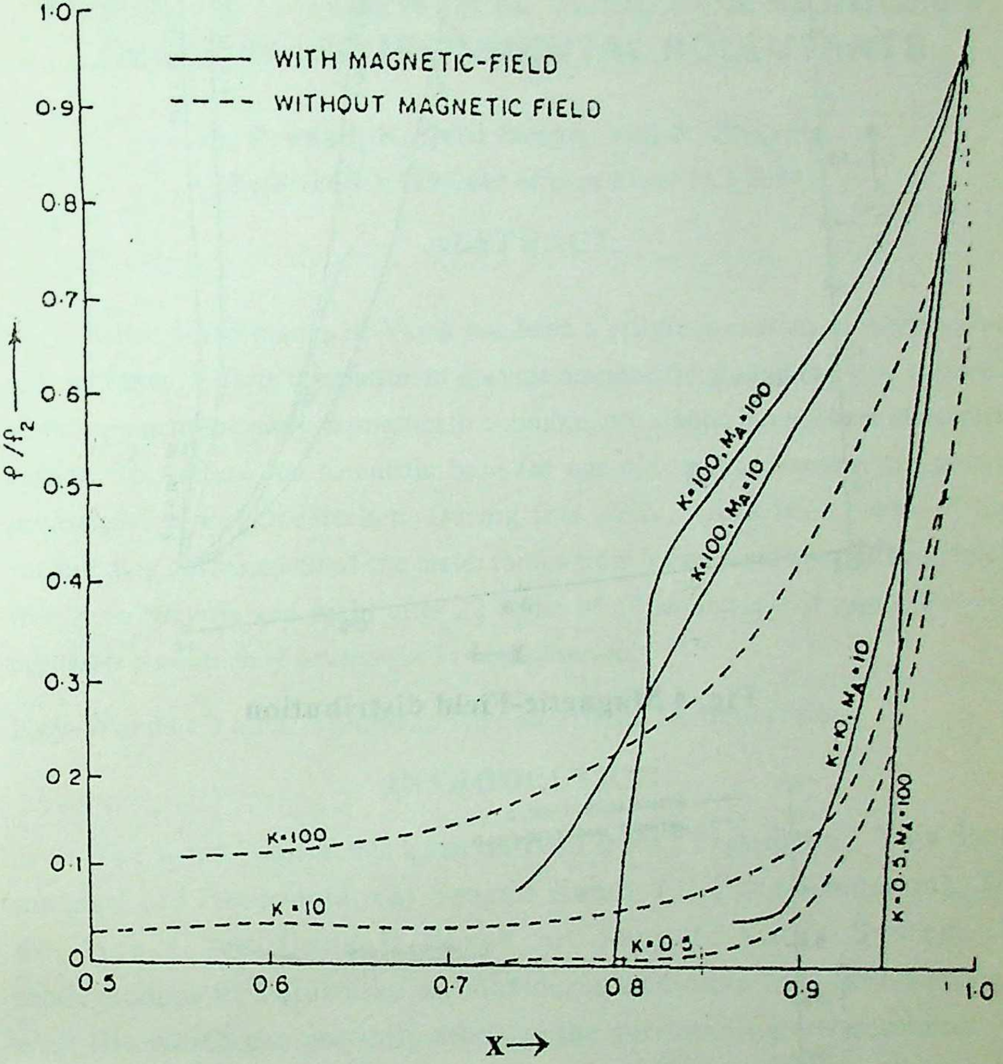

Fig. 3 Density distribution

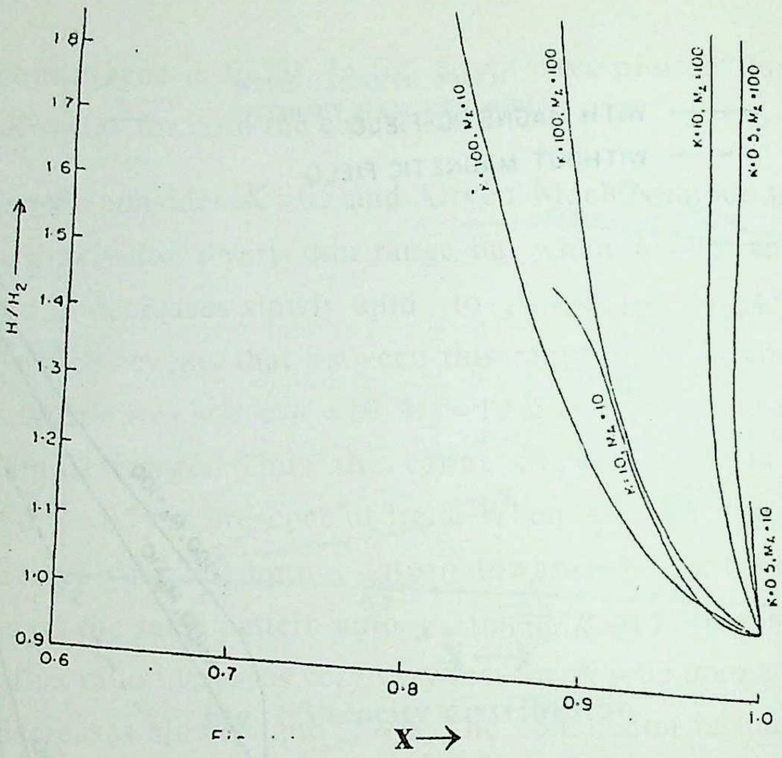

Fig. 4 Magnetic-Field distribution

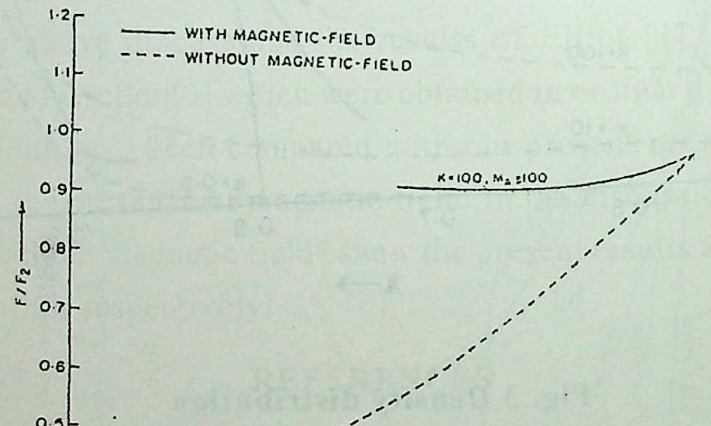

Fig. 5 Radiation flux distribution

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YAJNA AS A SCIENTIFIC TOOL FOR REDUCING CERTAIN ENVIRONMENTAL POLLUTANTS

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ABSTRACT

Albeit performance of Yajna has been a religious custom of Aryans since ancient times. Efforts in relation to provide a scientific ground to yajna as purfier of environment specially in relation to microbial population has not been considered earlier. To achieve the scientific base for our old vaidic customs, the present investigation was undertaken. During this study, it has been recorded that surrounding environment of one meter radius from Yajna kund became completely free from bacteria and fungi after 22 hours of of performance of yajana. However negligible population of actinomycetes were observed.

Key-Words : Yajna, Agnihotra (fire sacrifice) Aeromicroflora.

INTRODUCTION

The word 'Yajna' has been derived from "Yaja-dhatu". Yaja dhatu means : (1) Devapooja, (2) Sangati Karan, (3) Dana (Donation). The devapooja and Dana is based on Sangati Karan System. In modernconcept, Yajna may be considered as simple fumigation process with fire which can partially sterilize the surrounding environment.

The revival of Yajna popularity appeared in the country with Swami Dayanand Saraswati. In ancient time, Rishis used to perform Yajna early in the morning. Ancient literature shows that there are 5 main types of Yajna common in India such as : (1) Bhrama Yajna. (2) Deva Yajna, (3) Pitra Yajna, (4) Atithi Yajna, and (5) Balivaisvadeva Yajna.

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Vedas are believed to be very ancient, sacred and scientific record written in sanskrit script. Vedas deal various disciplines of science whcih are equally relevants to modern scientific knowledge related with service of mankind. Yajna are called the universal physicians by vedas as the Yajna are doubly advantageous to mankind as they restore normally to health and hygiene and longevity to the people of world. V.S. Vedashrami (1).

Maharishi Dayanand stated that Agnihotra is an efficient method to keep the environment clean and unpolluted. The Chapter-18 of Yajurveda states that Yajna is capable to provide all kinds of requirements (i.e. economic and spiritual) to the mankind as cleared by following hymn:

ऋतं च मेऽमृतं च मेऽयक्ष्मं च मेऽनामयच्च मे जीवातुश्च मे दीर्घायुत्वं च मेऽनमित्रं च मेऽभयं च मे सुखं च मे शयनं च मे सूषाश्च मे सुदिनं च मे यज्ञेन कल्पन्ताम्।। (शुक्लयजुर्वेदसंहिता १८.६)

Yajna has also believed to be useful in curing from fatal diseases as well as in keeping good health, as it is cleared by following Richa from Atharveda :

मुञ्चामित्वा हविषः जीवनाय कमज्ञातयक्ष्मादुत राजयक्ष्मात्। (अथर्ववेद संहिता ३.११.१)

Above richa (hymn) means that "I cure the patients from all known and unknown diseases by offering oblation in the sacred fire".

अयक्ष्मं मे अनामयच्चमे यज्ञेनकल्पन्ताम्। (शु०य०वे० १८.६)

Above richa indicates that "Yajna helps in dispelling the deadly diseases like tuberculosis". Various human diseases can be controlled by regular performance of Yajna system which is also called as Yajna therapy.

In the light of above view, no doubt about the importance of Yajna in various aspects of human welfare including health care. But scientific studies have not been carried out to provide scientific background for Yajna's role in purification of environment. Therefore present investigation was undertaken to prove the Yajna as a scientific tool for purification of environment specially in relation to aeiromicroflora under laboratory conditions.

MATERIALS & METHODS

(A) Materials :

Following materials were used during performance of Yajna :

(1) Yajna Kund, (2) Large spoon or Sruka, (3) Panchpatra, (4) Achmani, (5) Bowl, (6) Large plates, (7) Sets of Musala, (8) Adhardhari and Uttardhani, (9) Antardhana, (10) Sadavata.

Yajna Kund is generally made up of either copper or iron, Size of Yajna kund is (1xbxh = 12"x3"x12")

(B) Fumigation Substances :

1. Wood : Wood which is used in Yajna should be neat and clean, dry and free from insects. Following types of wood are recommended for Yajna performance consists of Pipal, Bargad, Mango, Neem etc. Out of these mango is most common and wedely used in the Yajna.

2. Ghee : Ghee (a boiled form of butter) is needed to provide fuel for keeping the flame constant. One spoon is used for each Ahuti. Traditionally, ghee derived from cow milk is used in yajnas.

3.Hawan Samagri : Ingredients of Hawan Samagri-

One Kg. Hawan Samgri contained the following twenty five ingredients

Chandan	- Santalum album	- 60g
Agra	- Aqularia malaccenis	-40g
Gugal	- Boswellia serrata	- 80g
Jaifal	- Myristica fregrens	- 1 piece
Javitri	-	- 1
Coconut	- Cocos nusifera	- 80g
Elaichi	- 1 lettariaerdam-ommum	-20g
Nagar motha	- Cyperus scariosus	- 80g
Tamal patra	- Cassia cinnaumon	- 40g
Panadi	-	- 40g
Dal chinee	-	- 20g
Tagar	- Valeriana wal	- 40g
Chhuare	-	- 8 pieces
Barley	-	- 40g
Awala	-	- 4 pieces
Kishmish	-	- 80g
Kakhor Kachri	- Hedy chium	- 40g
Nag Keshar	- Crocus sativus	- 20g
Tumbur	-	- 70g
Supari	-	- 1 piece
Neem Leaves	-	- 5
Reetha	-	- 10g
Laung	- Carypphiyllus aromatius	- 20g
Chirongi	- Bucbanania latifolia	- 10g
Til	- Sesamumindicum	- 20g
Sweet (sugar or shakkar)	-	- 200g

(C) Media :

Three different types of media were used for enumeration of microflora during performance of Yajana·

(1) Nutrient Agar Medium (For Bacteria)

(2) Czapekdox Agar Medium (For Fungi)

(3) Knight Agar Medium (For Actinomycetes)

Different types of media were poured in sterilized petridishes and these plates were exposed at different time for 5 min. (Before Yajna, Yajna flaming, Peak Yajna, at the end of Yajna and next day of Yajna). Three replicates were used in each case. Exposure of the plates was made at two different distances i.e. 3 & 3.5 ft. from Yajna kund.

Exposed plates were incubated at 37°C for for bacteria and 30°C for fungi in BOD incubator. Bacterial colony were counted after 24 hrs. and fungi after 72 hrs. of incubation period.

RESULTS

Results obtained during this investigation have been presented in the Table-1. Quantitative enumeration of microflora of the air under the influence of the Yajna has shown the presence of various group of micro organisms i.e. Bacteria, Fungi and Actinomycetes. Among these bacteria were the most dominating group of microorganisms at different stages of completion of the Yajna.

Maximum bacterial population was recorded at the time of starting of Yajna and no bacterial population was found on the next day i.e. about 22 hrs. of copletion of Yajna. This trend was also found in case of fungal organisms.

Variation in temperature and humidity was also found in the begining of Yajana. Minimum temperature recorded was 21°C and maximum 28°C at the peak Yajana. Similarly, maximum humidity was 70% and minimum was 38%.

DISCUSSION

Quantitative enumeration of microflora of air under influence of Yajna has shown great variability among different groups of microorganisms i.e. bacteria, fungi, and actinomycetes. This trend is in accordance to soil microflora and it may be possible due to adhered soil particles with samidha and further their multiplication with in the samidha during starage period. Bacteria were found to be the most dominating group among all the microorganisms. Number of bacterial colonies were enhanced during the beginning of Yajna (at flaming time of Ahuti).

There is increase in bacterial population at second sampling time which may be due to distribution of small particles of various matter of samidha which are not burnt completely in Yajana fire and with the flame and air were spread in the surrounding atmosphere and this may be perhaps a major cause of enhancement of microbial population at the time of second sampling. Total microbial profile was decreased at the Peak of Yajana. which may be due to increased temperature around 150°C in the kund and complete burning of various contents such as oils, esters, cellulose, thymol etc. of samidha.

Data recorded after 22h of completion of the Yajana indicated that slowly and gradually the system has been found effective to control entire group of microbes. Further it is recommended that Yajana should be performed at least three times more to ascertain the above findings in relation to its role in partial sterilization of the environment in terms of air microflora.

TABLE No.-1

QUANTITATIVE ENUMERATION OF MICROFLORA OF AIR UNDER INFLUENCE OF YAJNA

S. No.	Time of Exposure	Stages of Yajna	Temperature	Humidity	Average No. of Bacterial Colonies	Average No. of Fungal Colonies	Average No. of Actinomycetes colonies
1.	10:25 AM	Before Yajna	22^0C	67	08	08	08
2.	10:45 AM	Yajna Flaming	22^0C	65	18	08	01
3.	11:05AM	Peak Yajna	26^0C	45	06	01	0
4.	11:30AM	Yajna End	28^0C	38	03	01	01
5.	08:30 AM	Next Day (After 22 hrs.)	21^0C	70	0	0	01

Number of Replicates used : Three replicates were used in each case of organism at different stages of Yajna.

REFERENCE

1. Veer Sen Vadashrami : 'Yajana', "A foremost excellent project for universal florishment" (a translation of elven essays on the science of Yajana), published by Ved Sadan, Maharani Path, Indore (1962) 1-107.